(101MA24)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, DEC 2025
First Semester
Mathematics
ALGEBRA
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define a group and provide two non-trivial examples. Prove that the inverse of the inverse of an element is the element itself.

   (b) Prove that a subset H of a group G is a subgroup if and only if it is non-empty and for all $a, b \in H$, $ab^{-1} \in H$.

2. (a) Define a homomorphism between two groups. Prove that the kernel of a group homomorphism is a normal subgroup.

   (b) Prove that the image of a group homomorphism is a subgroup of the codomain group.

3. (a) Define permutation group and illustrate S_3 with all its elements and their composition table.

   (b) State and prove Cayley's theorem. Give an example of a group isomorphic to a permutation group.

4. (a) State and prove the First Sylow Theorem.

   (b) A group GGG has order 30. Determine the possible number of Sylow-5 subgroups and show your work.

5. (a) Define a ring with two examples. Show that Z_n commutative ring with unity.

   (b) Define an ideal. Prove that the kernel of a ring homomorphism is an ideal.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, DEC 2025
First Semester
Mathematics
ALGEBRA
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Prove that every finite abelian group is isomorphic to a direct product of cyclic groups of prime power order.

   (b) Show that $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$

2. (a) Define Euclidean domain and show that Z is a Euclidean domain.

   (b) Show that every Euclidean domain is a Principal Ideal Domain (PID).

3. (a) Let $f(x) = x^3 + 2x^2 - x - 2 \in Q[x]$. Factor $f(x)$ over Q.

   (b) Define polynomial ring. Prove that $Q[x]$ is a Euclidean domain.

4. (a) Define a vector space over a field with an example. Show that the set of all ordered pairs $(x, y) \in R^2$ is a vector space over R.

   (b) Prove that every finite-dimensional vector space has a basis.

5. (a) Define dual space. Prove that $\dim(V^*) = \dim(V)$ when V is finite-dimensional.

   (b) Let $V = R^3$. Find the dual basis corresponding to the standard basis.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

ANALYSIS – I

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Define upper and lower limits of a sequence. Show that every bounded sequence has a convergent subsequence.

   (b) Prove that if a series is absolutely convergent, then it is convergent.

2. (a) State and prove the Ratio Test for convergence.

   (b) Test the convergence of the power series:

   $\sum_{n=1}^{\infty} \frac{x^n}{n}$ for different values of *xxx*.

3. (a) Prove that a continuous function on a connected set takes all intermediate values (Intermediate Value Theorem).

   (b) Show that a continuous function on a closed interval is bounded and attains its maximum and minimum.

4. (a) Define infinite limit and limit at infinity. Show that $\lim_{x \to \infty} \frac{1}{x} = 0$.

   (b) Discuss the types of discontinuities with examples.

5. (a) Prove that if *f* has a local maximum at *ccc*, and is differentiable at *c*, then $f'(c) = 0$.

   (b) State and prove Rolle's Theorem with geometric interpretation.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

ANALYSIS – I

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Show that if a function f is twice differentiable and $f'(x) > 0$, then f is convex.

   (b) Use Taylor's theorem to approximate e^x at $x = 0$. Using third-degree polynomial and estimate the error.

2. (a) Define and prove the linearity of Riemann-Stieltjes integral.

   (b) Evaluate $\int_1^3 x^2 d[x]$, where $[x]$ denotes the greatest integer function.

3. (a) Let $r(t) = (e^t, \ln t)$. Find $r'(t)$.

   (b) Discuss the continuity of vector-valued functions and state conditions for differentiability.

4. (a) Prove that integration and differentiation can be interchanged under certain conditions.

   (b) Let $f(x) = x^2$. Find the arc length of f from $x = 0$ to $x = 1$.

5. (a) Let $r(t) = (t, t^3), 0 \le t \le 1$. Compute the integral $\int_0^1 r(t)\,dt$.

   (b) Define a rectifiable curve and prove that a differentiable curve with continuous derivative is rectifiable.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define a linear differential equation of first order. Give an example.

   (b) Solve the equation :

   $$\frac{dy}{dx} + 2y = \sin x$$

2. (a) Define homogeneous second order linear differential equation with constant coefficients.

   (b) Solve the differential equation.

   $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

3. (a) Define linear dependence and independence of solutions.

   (b) Compute the Wronskian of

   $y_1(x) = e^x$ and $y_2(x) = xe^x$.

4. (a) Explain the wronskian and its significance in determining linear independence.

   (b) Show that the wronskian of two linearly independent solution is non-zero.

5. (a) Define a second-order linear differential equation with variable coefficients.

   (b) Solve the equation using reduction of order:

   Given one solution $y_1 = x$, find the general solution of $x^2y'' - 3xy' + 4y = 0$.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Describe the method of reduction of order solving a second solution.

   (b) Solve $x^2y'' + xy' - y = 0$.

2. (a) Define regular singular point.

   (b) Solve the Euler equation:

   $x^2y'' - 3xy' + 4y = 0$

3. (a) What is a regular singular point? Explain with example.

   (b) Solve the equation using Frobenius method:

   $x^2y'' + xy' + (x^2 - 1)y = 0$.

4. (a) Define an exact differential equation.

   (b) Determine whether the equation is exact and solve:

   $(2xy + y^3)dx + (x^2 + 3y^2x)dy = 0$.

5. (a) Define and give an example of variables separable equation.

   (b) Solve $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

TOPOLOGY

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Define a metric space. Give two examples that are not subsets of R^n.

   (b) Show that a continuous function $f:(X,d)\to(Y,\rho)$ maps convergent sequences to convergent sequences.

2. (a) Prove that the open ball $B(x,r)$ is an open set in a metric space.

   (b) Show that R with the usual metric is a complete space.

3. (a) Define a topological space. Give two non-metric examples.

   (b) Prove that any union of open sets is open in a topological space.

4. (a) State the axioms that define a topology.

   (b) Let $X = \{a,b,c\}$. List all topologies on X.

5. (a) State and explain the Ascoli-Arzelà theorem.

   (b) State and prove Tychonoff's theorem for finite products.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

TOPOLOGY

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Give an example of a compact space which is not sequentially compact.

   (b) Prove that every compact metric space is complete and totally bounded.

2. (a) Define T_0, T_1 and T_2 (Hausdorff) spaces with examples.

   (b) Prove the Tietze Extension Theorem for normal spaces.

3. (a) Give an example of a space that is T_1 but not T_2.

   (b) Prove that the real line R with usual topology is normal.

4. (a) Define connectedness and show that the image of a connected space under a continuous map is connected.

   (b) State and prove the Urysohn Embedding Theorem.

5. (a) Prove that the continuous image of a connected space is connected.

   (b) Let X be a compact Hausdorff space. Show how it can be embedded in a cube.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

ADVANCED DISCRETE MATHEMATICS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Define tautology, contradiction and contingency with examples.

   (b) Construct a truth table and determine whether the following formula is a tautology $(P \to Q) \vee (\neg P)$.

2. (a) Explain the concept of duality law in prepositional logic.

   (b) Reduce the statement

   $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ and verify whether it is a tautology.

3. (a) What is consistency of premises? Explain with an example using the indirect method of proof.

   (b) Convert the following into prenex normal form and identify free and bound variables:

   $\forall x(P(x) \to \exists y\, Q(y,x))$.

4. (a) State and explain rules of inference in predicate logic.

   (b) Using rules of inference, derive the conclusion :

   Given :

   (i) $\forall x(P(x) \to Q(x)$

   (ii) $P(a)$

   Prove $Q(a)$.

5. (a) Define a finite state machine. Explain with an example how to construct a state table and diagram.

   (b) Given a state table for a machine with three states A, B, C and input symbols {0, 1}, analyze the behavior and identify reachable states from A.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Mathematics

ADVANCED DISCRETE MATHEMATICS
MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Differentiate between Mealy and Moore machines with suitable diagrams.

   (b) Construct the state diagram for a machine that accepts binary strings ending with '01'.

2. (a) Define a distributive lattice. Prove that every Boolean algebra is a distributive lattice.

   (b) Let L={1,2,3,6} under divisibility. Determine whether LLL forms a lattice. Draw its Hasse diagram.

3. (a) Prove or disprove: Every lattice is a Boolean algebra.

   (b) Express the Boolean polynomial

   $F(x, y, z) = xy + x'z + yz$ in minimal form.

4. (a) Define ideals and filters in lattices. Give examples.

   (b) Construct a switching circuit for the Boolean expression $F = AB + A'C$ and simplify it using Boolean algebra.

5. (a) Explain the concept of minimal forms of Boolean expressions using Karnaugh maps.

   (b) Simplify the Boolean function F(A,B,C)= Σ(1,3,5,7) using a Karnaugh map and draw the resulting switching circuit.
