201MA24

ASSIGNMENT - 1
M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper I – GALOIS THEORY

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. Let $F \subseteq E \subseteq K$ be field. If $[K : E] < \infty$ and $[E : F] < \infty$, then show that

   (i) $[K : F] < \infty$

   (ii) $[K : F] = [K : E][E : F]$

2. State and Prove Gauss Lemma.

3. If E is an extension of F and $u \in E$ is algebraic over F, then prove that $F(u)$ is an algebraic expansion of F.

4. State and Prove Kronecker theorem.

5. Prove that for any field K the following are equivalent.

   (a) K is algebraically closed,

   (b) Every irreducible polynomial in $K[x]$ is of degree 1,

   (c) Every polynomial in $K[x]$ of positive degree factor completely in $K[x]$ into linear factors,

   (d) Every polynomial in $K[x]$ of positive degree has atleast one root in K.

6. (a) If $f(x) \in F[x]$ is irreducible over F, then show that all roots of $f(x)$ have the same multiplicity.

   (b) State and prove uniqueness of splitting field theorem.

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ASSIGNMENT - 2
M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper I – GALOIS THEORY

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. Show that if E is a finite separable extension of a field F, then E is a simple extension of F.

2. (a) State and prove Dedekind lemma.

   (b) Let H be a finite subgroup of the group of automorphisms of a field E. Then show that $[E : E_H] = |H|$.

3. State and prove fundamental theorem of algebra

4. (a) Let F be a field let U be a finite subgroup of the multiplicative group $F^* = F - \{0\}$. Then show that U is cyclic.

   (b) Show that $\emptyset_n(x) = \pi_\omega(x - \omega)$, ω is primitive n^{th} root in C, is an irreducible polynomial of degree $\emptyset(n)$ in $Z[x]$.

5. (a) Show that $f(x) \in F[x]$ is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group $G(E/F)$.

   (b) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals over Q.

6. (a) Solve the problem of trisecting an angle.

   (b) Prove that it is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only.

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Second Semester

Mathematics
Paper II – Analysis – II

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. a) Let $\{f_n\}$ be a sequence of functions defined on E. Then the sequence $\{f_n\}$ convergence uniformly on E if and only if for every $\varepsilon > 0$ there exist an integer N such that $m \geq N, n \geq N, x \in E$ implies $|f_n(x) - f_m(x)| \leq \varepsilon$.

b) Let (x, d) be a metric space and $E \subseteq X$. If $\{f_n\}$ is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then f is continuous on E.

2. Suppose K is a compact subset of a metric space (x, d), and

a) $\{f_n\}$ is a sequence of continuous functions on K,

b) $\{f_n\}$ converges pointwise to a continuous function of on K

c) $f_n(x) \geq f_{n+1}(x)\ \forall\ x \in K, n = 1,2,3\ldots$

Then $f_n \to f$ uniformly on K.

3. Show that there exists a real continuous function on the real line which is nowhere differentiable.

4. Prove that a subset S of $\mathbb{C}(K)$ is compact if and only if it is uniformly closed, pointwise bounded, and equicontinuous, where K is a compact metric space.

5. Let B be the uniform closure of an algebra A of bounded functions, then show that B is an uniformly closed algebra.

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M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper II – Analysis – II

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. Prove that
   a) The function E is periodic, with period $2\pi i$.
   b) The functions C and S are periodic, with period 2π
   c) If $0 < t < 2\pi$, then $E(it) \neq 1$
   d) If z is a complex number with |z|=1, there is an unique t in $[0,2\pi]$ such that $E(it) = t$
2. Suppose X is a vector space, and $\dim X = n$.
   (a) A set E of n vectors in X spans X if and only if E is independent.
   (b) X has a basis, and every basis consists of n vectors.
   (c) If $1 \leq r \leq n$ and $\{\bar{y}_1, \dots, \bar{y}_r\}$ is an independent set in X, then X has a basis Containing$\{\bar{y}_1, \dots, \bar{y}_r\}$.
3. suppose E is an open set in R^n, $\bar{f}$ maps E into $R^m, \bar{f}$ is differentiable at $\bar{x}_0 \in E, \bar{g}$ maps an open set containing $\bar{f}(E)$ into R^k, and $\bar{g}$ is differentiable at $\bar{f}(x_0)$. Then the mapping F of E into R^k defined by $\bar{F}(\bar{x}) = \bar{g}\left(\bar{f}(\bar{x})\right)$ is differentiable at $\bar{x}_0$, and $\bar{F}'(x_0) = \bar{g}'\left(\overline{f'}(x_0)\right)\bar{f}'(\bar{x}_0)$
4. a) : If $A \in L(\mathbb{R}^{n+m}, \mathbb{R}^n)$ and if $A_{\boldsymbol{x}}$ is invertible, then there corresponds to every $\mathbf{k} \in \mathbb{R}^m$ a unique $\boldsymbol{h} \in \mathbb{R}^n$ such that $A(\boldsymbol{h}, \boldsymbol{k}) = \mathbf{0}.$ This $\boldsymbol{h}$ can be computed from $\boldsymbol{k}$ by the formula $\boldsymbol{h} = -(A_{\boldsymbol{x}})^{-1}A_{\boldsymbol{y}}\mathbf{k}$
   b) A linear operator A on $\mathbb{R}^n$ is invertible if and only if $\det[A] \neq 0$
5. Suppose f is defined in an open set $E \subseteq \mathbb{R}^2$, and $D_1 f$ and $D_{21} f$ exist at every point of E. Suppose $Q \subseteq E$ is a closed rectangle with sides parallel to the coordinate axes, having (a, b) and $(a + h,\ b + k)$ as opposite vertices $(h \neq 0,\ k \neq 0)$.
   put $\varDelta(f,\ Q) = f(a + h,\ b + k) - f(a + h,\ b) - f(a,\ b + k) + f(a,\ b)$.
   Then there is a point (x, y) in the interior of Q such that
   $\Delta(f,\ Q) = hk(D_{21}f)(x,\ y)$.

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Second Semester

Mathematics
Paper III - Measure and Integration

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. a) If A and B are any two subsets of R such that$A \subseteq B$, then $m^*A \leq m^*B$.

   b) Let $\{A_n\}$ be a countable collection of sets of real numbers.

   Then show that $m^*(U_n A_n) \leq \sum_n m^*(A_n)$.

2. a) Let $\subseteq [0,1\ )$ be a measurable set. Then for each $\in [0,1\ )$ the set $E\dot{+}y$ is also measurable and $m(E\dot{+}y) = mE$.

   b) Any continuous real valued function defined on a measurable set is measurable.

3. Let ϕ and ψ be two simple functions which vanish outside a set of finite measure then

   i) $\int (a\phi + b\psi) = a\int \phi + b\int \psi$ ii) $\phi \geq \psi$ almost everywhere then $\int \phi \geq \int \psi$

4. Let g be a integrable over E and let $\{f_n\}$ be a sequence of measurable functions such that $|f_n| \leq g$ on E and for almost all x in E we have $f(x) = \lim f_n(x)$ .then $\lim \int_E f_n = \int_E f$.

5. Let E be a set of finite outer measure and **I** a collection of intervals that cover E in the sense of Vitali. Then, given $\varepsilon > 0$, there is a finite disjoint collection $\{I_1, I_2, \ldots I_N\}$ of intervals in **I** such that $m^*[E \sim \bigcup_{n=1}^{N} I_N] < \varepsilon$.

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M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper III - Measure and Integration

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. a) If f is a function of bounded variation on [a,b] then $T_a^b = P_a^b + N_a^b$ and

   $f(b) - f(a) = P_a^b - N_a^b$.

   b) A function f is bounded variation on [a,b] iff f is difference of two monotonic real valued functions on [a,b].

2. (a) Let ϕ has a second derivative at each point of (a, b). Then ϕ is convex on $(a, b) \Leftrightarrow \phi''(x) \geqslant 0\ \forall x \in (a, b)$.

(b) Let ' ϕ ' be a convex function on $(-\infty, \infty)$ and f be integrable function on $[0,1]$. Then

$$\int_0^1 \phi(f(t))dt \geqslant \phi\left(\int_0^1 f(t)dt\right).$$

3. a) Prove that $\|f + g\|_\infty = \|f\|_\infty + \|g\|_\infty$.

   b) If $f \in L'$ and $g \in L^\infty$ then prove that $\int_0^1 |fg| \leqslant \|f\|_1 \|g\|_\infty$

4. a) A normed linear space X is complete if and only if every absolutely summable series is summable.

   b) Prove that l^p is complete $(1 \leq p < \infty)$.

5. Show that L^∞ is completeness of $C([a, b])$.

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M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025
Second Semester
Mathematics
Paper IV - INTEGRAL EQUATIONS
MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. (a) Show that the function, $\varphi(x) = \frac{1}{(1+x^2)^{\frac{3}{2}}}$ is a solution of the Volterra integral equation $\varphi(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} \varphi(t)dt$.
   (b) With the aid of the resolvent kernel, find the solution of the integral equation, $\varphi(x) = e^{x^2} + \int_0^x e^{x^2-t^2} \varphi(t)dt$.
2. (a) Using the method of successive approximations, solve the integral
$$\varphi(x) = x - \int_0^x (x - t)\,\varphi(t)\,dt, \quad \varphi_0(x) = 0.$$
   (b) Solve the system of integral equations,
$$\varphi_1(x) = 1 - 2\int_0^x e^{2(x-t)}\varphi_1(t)dt + \int_0^x \varphi_2(t)dt,$$
$$\varphi_2(x) = 4x - \int_0^x \varphi_1(t)dt + 4\int_0^x (x - t)\,\varphi_2(t)dt.$$
3. (a) Solve the following integro-differential equations:
$$\varphi''(x) + \int_0^x e^{2(x-t)}\,\varphi'(t)dt = e^{2x};\ \varphi(0) = 0, \varphi'(0) = 1,$$
   by using the Laplace Transformation.

   (b) Solve the integral equation, $\int_0^x (2 + x^2 - t^2)\varphi(t)dt = x^2$

   by the method of successive approximations.

4. (a) Show that, $\int_{-1}^1 (1 + x)^{p-1}(1 - x)^{q-1}\,dx = 2^{p+q-1}B(p, q)$.
   (b) Solve the integral equation, $\int_0^x \cosh(x - t)\varphi(t)dt = x$.

5. (a) Using the Fredholm determinants, find the resolvent kernels of the following kernel, $K(x,t)=x^2t-xt^2$; $0\leq x, t\leq 1$.

   (b) Construct the resolvent kernel for the following kernel
   $K(x, t)=(1+x)(1-t)$; $a=-1,\ b=0$.

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M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics

Paper IV - INTEGRAL EQUATIONS

MAXIMUM MARKS: 30

ANSWER ALL QUESTIONS

1. (a) Solve the integral equation $\varphi(x)=\lambda\int_0^1 xt\varphi^2(t)dt$, where λ is a parameter.

   (b) Find the characteristic numbers and eigenfunctions of the homogeneous equation

$$\varphi(x)-\lambda\int_0^1 K(x,t)\varphi(t)dt=0$$

where,

$$K(x,t)=\begin{cases} x(t-1), & 0\le x\le t, \\ t(x-1), & t\le x\le \pi. \end{cases}$$

2. (a) Solve the equation, $\varphi(x)$-$\lambda\int_0^\pi(\cos^2 x\cos 2t+\cos^3 t\cos 3x)\,\varphi(t)dt=0$.

   (b) Solve, $\varphi(x)$-$\lambda\int_0^1(5x^2$-$3)t^2\,\varphi(t)dt=e^x$.

3. (a) Find Green's function for the boundary-value problem

$$y''(x)+k^2y=0$$
$$y(0)=y(1)=0$$

   (b) Solve the boundary value problem using Green's function $y''+y=x$,

$$y(0)=y\left(\frac{\pi}{2}\right)=0$$

4. Reduce the boundary value problem $y''=\lambda y+x^2$, $y(0)=y\left(\frac{\pi}{2}\right)=0$ to the integral equation.

5. (a) Show that the integral equation $\varphi(x)=\lambda\int_0^\infty\varphi(t)\,sinxt\,dt$ has characteristic number $\lambda=\pm\sqrt{\frac{2}{\pi}}$ of infinite multiplicity and find the associated eigenfunctions.

   (b) Solve the integral equation by using the Bubnov-Galerkin method

$$\varphi(x)=1+\frac{4}{3}x+\int_{-1}^{1}(xt^2-x)\varphi(t)\,dt\,.$$

ASSIGNMENT - 1
M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper V - GRAPH THEORY

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1 (a) Explain the Puzzle with multi colored cubes (or the puzzle called "Instant Insanity Puzzle") along with a solution.

(b) Define connected graph and give an example of a connected graph.

Prove that: A graph G is disconnected $\Leftrightarrow$ its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G, whose one end-vertex is in V_1 and the other end vertex is in V_2.

2 (a) Define Euler Graph. Prove that a given connected graph G is an Euler graph $\Leftrightarrow$ all the vertices of G are of even degree.

(b) Prove the Theorem whose statement was given by: Let G be a graph and v be a vertex in G. An Euler graph G is arbitrarily traceable from the vertex v in G $\Leftrightarrow$ every circuit in G contains v.

3 (a) What do you mean by 'the seating arrangement problem with nine members', and Solve it.

(b) Prove the following theorem: For a given graph G with n vertices, the following conditions are equivalent: (i) G is connected and is circuitless; (ii) G is connected and has n -1 edges; (iii) G is circuitless and has n -1 edges; (iv) There is exactly one path between every pair of vertices in G; (v) G is a minimally connected graph; and (vi) G is a tree.

4 (a) Prove the following theorem: Let G be a connected graph. The distance d(v, u) between two vertices v and u is a metric.

(b) What do you mean by a binary tree. Explain three properties of binary trees.

5 (a) Prove that with respect to any of its spanning trees, a connected graph of 'n' vertices and 'e' edges has 'n-1' tree branches and e - n + 1 chords.

(b) Prove that: Stating from any spanning tree of a graph G, we can obtain every spanning tree of G by successive cyclic interchanges..

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M.Sc. DEGREE EXAMINATIONS, DECEMBER -2025

Second Semester

Mathematics
Paper V - GRAPH THEORY

MAXIMUM MARKS: 30
ANSWER ALL QUESTIONS

1. (a) Explain Kruskal's algorithm to find shortest spanning tree along with an example.

   (b) Prove that: Every circuit has an even number of edges in common with any cut-set.

2. (a) Define Vertex Connectivity, Edge Connectivity, and Separable graph. Give examples to each one of the three concepts.

   (b) Prove that: A vertex v in a connected graph G is a cut-vertex $\Leftrightarrow$ there exists two vertices x and y in G such that every path between x and y passes through v.

3. (a) Define planar graph, and non planar graph, give examples for each.

Explain the four Common properties of Kuratowski's 1^{st} and 2^{nd} graphs:

   (b) Show that: Kuratowski's 2^{nd} graph is a non-planar graph.

4. (a) Explain: A Procedure to construct a dual graph from a given graph with a suitable example..

   (b) Prove that: If G^* is the dual of G, then show that

   (i) rank of G = nullity of G^* [that is $r = \mu^*$]

   (ii) rank of G^* = nullity of G [that is $r^* = \mu$]

5. (a) Show that: The set consisting of all the circuits and the edge-disjoint unions of circuits (including the null set ϕ) in a graph G is an Abelian group under the operation ring sum $\oplus$.

   (b) Define modulo 3 arithmetic on the set $\mathbb{Z}_3$ = {0, 1, 2}, the set of integers modulo 3. Also write down the addition and multiplication tables with respect to binary operations addition modulo 3 and multiplication modulo 3.