M.Sc. DEGREE EXAMINATION, DEC 2025

First Semester

Statistics

PROBABILITY THEORY AND DISTRIBUTIONS MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

- 1. (a) Define limit supremum and limit infimum of sequence of sets. Prove that Lim $\inf A_n \subseteq Lim \mathrm{sup} A_n.$
 - (b) Prove that the probability measure is countably additive.
- 2. (a) Define field and σ -field. Give examples.
 - (b) Define statistical independence and prove that if $A \subset B$ and P(B) > 0, then $P(A \mid B) \le 1$.
- 3. (a) Define measurable function. Show that a composition of measurable functions is measurable.
 - (b) Define and explain Holder's inequality.
- 4. (a) Derive the inversion formula for the characteristic function.
 - (b) Define marginal and conditional distributions Give one example.
- 5. (a) Prove Kintchine's W.L.L.N for i.i.d. random variables with finite mean.
 - (b) Give an example to show that convergence in probability does not imply almost sure convergence.

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Statistics

PROBABILITY THEORY AND DISTRIBUTIONS MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

- 1. (a) Explain the concepts of convergence in distribution and almost sure convergence.
 - (b) Prove that convergence in mean square implies convergence in probability.
- 2. (a) Define multinomial distribution. Derive the expected value and variance of its components.
 - (b) Define and explain truncated Poisson distribution.
- 3. (a) Derive the moment generating function of Laplace distribution.
 - (b) Write the applications and properties of the logistic distribution.
- 4. (a) Derive the distribution function of the r-th order statistic.
 - (b) Obtain the joint p.d.f. of order statistics in the case of continuous distribution.
- 5. (a) Define range and derive its distribution for exponential distribution.
 - (b) Give two applications of order statistics in reliability theory.

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Statistics

STATISTICAL COMPUTING USING R MAXIMUM MARKS :30 ANSWER ALL QUESTIONS

- (a) Discuss the different operators in R (arithmetic, relational, logical, assignment). Write code examples.
 - (b) Explain assignment, relation and miscellaneous operators using in 'R'.
- 2. (a) What are factors in R? How are factor variables created and manipulated? Explain with gl() and other functions.
 - (b) Describe integer, raw, numeric, logical, complex. Data types in 'R'.
- 3. (a) Explain the creation and operations on matrices and lists in R with examples.
 - (b) Explain about writing data files from other files.
- 4. (a) How can data be read into and saved from R?
 - (b) Explain the use of read.csv(), write.table(), save(), load() and file.choose() functions.
- 5. (a) Explain the different types of loops in R with examples.
 - (b) Write an R program using for loop to compute factorial of a number.

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STATISTICAL COMPUTING USING R MAXIMUM MARKS :30 ANSWER ALL QUESTIONS

- 1. (a) How are functions defined and called in R?
 - (b) Write a function to compute correlation and another to perform a paired ttest.
- 2. (a) Describe the built-in R functions for normality tests (Shapiro-Wilk, Kolmogorov-Smirnov). Write code to test normality for a given dataset.
 - (b) Explain Wilcoxon Mann-Whitney U-test by using R.
- 3. (a) Describe R-Codes for Exponential.
 - (b) You are provided with the following data representing the number of successes observed in a series of experiments, where each experiment consists of 4 independent Bernoulli trials (i.e., n=4).

Number of	Observed
Successes (x)	Frequency
0	5
1	12
2	18
3	10
4	5

Task:

(i) Fit a binomial distribution to the given data by estimating the probability of success ppp from the data.

- (ii) Write R code to:
 - Estimate the value of p
 - Compute the expected frequencies for each number of successes
 - Perform a chi-square goodness-of-fit test
- (iii) Clearly report the chi-square test statistic, degrees of freedom, and p-value.
- (iv) Based on the p-value, comment on the goodness of fit of the binomial model to the observed data.
- 4. (a) Write-R code to create pie charts, bar charts, and Q-Q plots.
 - (b) How can you control and modify plot elements like axes, text, grid, and legend?
- 5. (a) Explain Built in R-syntax for one-way ANOVA and two-way ANOVA.

(b) Write about box Whisker plots, dot plot Scatter plots in R.

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First Semester

Statistics

ESTIMATION MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

- 1. (a) Define and explain the concepts of parameter, parametric space, statistic, estimator and standard error with suitable examples.
 - (b) State and prove the Fisher-Neyman Factorization Theorem.
- 2. (a) Explain the concept of sufficiency with examples. What is the difference between joint density and likelihood function?
 - (b) Define complete and minimal sufficiency. Give examples to illustrate.
- 3. (a) Define UMVUE. Explain the properties and regularity conditions for UMVUE.
 - (b) State and prove the Rao-Blackwell Theorem.
- 4. (a) What is the Cramer-Rao inequality? State the condition for equality and provide an example.
 - (b) Define consistency and efficiency of an estimator. Explain CAN and CAUN estimators.
- 5. (a) Derive the MLE for the parameter of exponential distribution.
 - (b) Explain the method of moments and compare it with MLE.
- 6. (a) Explain the concept of percentile estimation with an example.
 - (b) Define minimum chi-square and modified minimum chi-square methods.

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Statistics

ESTIMATION MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

- 1. (a) What is interval estimation? How are confidence intervals constructed using pivots?
 - (b) Explain the relationship between confidence intervals and hypothesis testing.
- 2. (a) Define prior and posterior distributions. Explain the concept of Bayes Estimator with an example.
 - (b) What is a loss function and risk function? Define and discuss Minimax Estimator.
- 3. (a) Define Type I and Type II censoring in the context of lifetime data analysis.
 - (b) The lifetimes (in hours) of 6 electronic components were tested in a life test experiment. The experiment was terminated at a pre-fixed time T=10 hours (Type I censoring). The following failure times (in hours) were observed:

The remaining 2 units did not fail before time 10 and are therefore right-censored at 10 hours.

Tasks:

Define Type I and Type II censoring clearly.

Assume that the lifetimes follow an exponential distribution with probability density function:

$$F(t;\lambda) = \lambda e^{-\lambda t}, t \geq 0$$

Obtain the maximum likelihood estimate (MLE) of the parameter λ using the given censored data.

- 4. (a) Discuss truncated distributions and their application in estimation.
 - (b) Explain the procedure of constructing confidence intervals using pivots for censored normal data.

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Statistics

SAMPLING THEORY MAXIMUM MARKS :30 ANSWER ALL QUESTIONS

- 1. (a) Describe the method of stratified sampling. Derive the expression for its variance.
 - (b) Explain gain in precision due to stratification with an example.
- 2. (a) Discuss systematic sampling for populations with linear trend.
 - (b) Explain Yates end correction in systematic sampling.
- 3. (a) Define PPS sampling. Explain the method of selection and estimation with replacement.
 - (b) Describe Murthy's estimator for a sample of size two.
- 4. (a) Explain Horvitz-Thompson estimator and derive its variance.
 - (b) Discuss Midzuno-Sen sampling scheme.
- 5. (a) Explain two-stage sampling with equal number of second-stage units and derive the estimate of population mean.
 - (b) Derive the expression for variance and its estimation in two-stage sampling.

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SAMPLING THEORY MAXIMUM MARKS :30 ANSWER ALL QUESTIONS

- 1. (a) Define double sampling for stratification. Derive the expression for variance of the estimated mean.
 - (b) Explain optimum allocation in double sampling.
- 2. (a) Discuss double sampling for regression estimation and obtain the regression estimator.
 - (b) Explain sources and types of non-sampling errors.
- 3. (a) Describe the Hansen and Hurwitz technique for non-response adjustment.
 - (b) Explain Deming's model for non-sampling errors.
- 4. (a) Define ratio estimator. Derive its bias and mean square error.
 - (b) Compare ratio estimator with mean per unit estimator.
- 5. (a) Explain difference estimator and derive its variance.
 - (b) Compare regression estimator with ratio and mean per unit estimators.
