M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Mathematics

ALGEBRA

Time : Three hours Maximum : 70 marks

Answer ONE question from each unit.

All questions carry equal marks.

UNIT I

1. (a) Define a group and provide two non-trivial examples. Prove that the inverse of the inverse of an element is the element itself.

   (b) Prove that a subset H of a group G is a subgroup if and only if it is non-empty and for all $a, b \in H$, $ab^{-1} \in H$.

Or

2. (a) Define a homomorphism between two groups. Prove that the kernel of a group homomorphism is a normal subgroup.

   (b) Prove that the image of a group homomorphism is a subgroup of the codomain group.

UNIT II

3. (a) Define permutation group and illustrate S_3 with all its elements and their composition table.

   (b) State and prove Cayley's theorem. Give an example of a group isomorphic to a permutation group.

Or

4. (a) State and prove the First Sylow Theorem.

   (b) A group GGG has order 30. Determine the possible number of Sylow-5 subgroups and show your work.

UNIT III

5. (a) Define a ring with two examples. Show that Z_n commutative ring with unity.

   (b) Define an ideal. Prove that the kernel of a ring homomorphism is an ideal.

Or

6. (a) Prove that every finite abelian group is isomorphic to a direct product of cyclic groups of prime power order.

   (b) Show that $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$

UNIT IV

7. (a) Define Euclidean domain and show that Z is a Euclidean domain.

(b) Show that every Euclidean domain is a Principal Ideal Domain (PID).

Or

8. (a) Let $f(x) = x^3 + 2x^2 - x - 2 \in Q[x]$. Factor $f(x)$ over Q.

(b) Define polynomial ring. Prove that $Q[x]$ is a Euclidean domain.

UNIT V

9. (a) Define a vector space over a field with an example. Show that the set of all ordered pairs $(x, y) \in R^2$ is a vector space over R.

(b) Prove that every finite-dimensional vector space has a basis.

Or

10. (a) Define dual space. Prove that $\dim(V^*) = \dim(V)$ when V is finite-dimensional.

(b) Let $V = R^3$. Find the dual basis corresponding to the standard basis.

(102MA24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Mathematics

ANALYSIS – I

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

(5 × 14 = 70)

UNIT I

1. (a) Define upper and lower limits of a sequence. Show that every bounded sequence has a convergent subsequence.

   (b) Prove that if a series is absolutely convergent, then it is convergent.

Or

2. (a) State and prove the Ratio Test for convergence.

   (b) Test the convergence of the power series:

   $\sum_{n=1}^{\infty} \frac{x^n}{n}$ for different values of *xxx*.

UNIT II

3. (a) Prove that a continuous function on a connected set takes all intermediate values (Intermediate Value Theorem).

   (b) Show that a continuous function on a closed interval is bounded and attains its maximum and minimum.

Or

4. (a) Define infinite limit and limit at infinity. Show that $\lim_{x\to\infty} \frac{1}{x} = 0$.

   (b) Discuss the types of discontinuities with examples.

UNIT III

5. (a) Prove that if f has a local maximum at *ccc*, and is differentiable at c, then $f'(c) = 0$.

   (b) State and prove Rolle's Theorem with geometric interpretation.

Or

6. (a) Show that if a function f is twice differentiable and $f'(x) > 0$, then f is convex.

(b) Use Taylor's theorem to approximate e^x at $x = 0$. Using third-degree polynomial and estimate the error.

UNIT IV

7. (a) Define and prove the linearity of Riemann-Stieltjes integral.

(b) Evaluate $\int_1^3 x^2 d[x]$, where $[x]$ denotes the greatest integer function.

Or

8. (a) Let $r(t) = (e^t, \ln t)$. Find $r'(t)$.

(b) Discuss the continuity of vector-valued functions and state conditions for differentiability.

UNIT V

9. (a) Prove that integration and differentiation can be interchanged under certain conditions.

(b) Let $f(x) = x^2$. Find the arc length of f from $x = 0$ to $x = 1$.

Or

10. (a) Let $r(t) = (t, t^3), 0 \le t \le 1$. Compute the integral $\int_0^1 r(t)\,dt$.

(b) Define a rectifiable curve and prove that a differentiable curve with continuous derivative is rectifiable.

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(103MA24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

(5 × 14 = 70)

UNIT I

1. (a) Define a linear differential equation of first order. Give an example.

(b) Solve the equation :

$$\frac{dy}{dx} + 2y = \sin x$$

Or

2. (a) Define homogeneous second order linear differential equation with constant coefficients.

(b) Solve the differential equation.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

UNIT II

3. (a) Define linear dependence and independence of solutions.

(b) Compute the Wronskian of

$y_1(x) = e^x$ and $y_2(x) = xe^x$.

Or

4. (a) Explain the wronskian and its significance in determining linear independence.

(b) Show that the wronskian of two linearly independent solution is non-zero.

UNIT III

5. (a) Define a second-order linear differential equation with variable coefficients.

(b) Solve the equation using reduction of order:

Given one solution $y_1 = x$, find the general solution of $x^2y'' - 3xy' + 4y = 0$.

Or

6. (a) Describe the method of reduction of order solving a second solution.

(b) Solve $x^2y'' + xy' - y = 0$.

UNIT IV

7. (a) Define regular singular point.

(b) Solve the Euler equation:

$$x^2y'' - 3xy' + 4y = 0$$

Or

8. (a) What is a regular singular point? Explain with example.

(b) Solve the equation using Frobenius method:

$$x^2y'' + xy' + (x^2 - 1)y = 0.$$

UNIT V

9. (a) Define an exact differential equation.

(b) Determine whether the equation is exact and solve:

$$(2xy + y^3)dx + (x^2 + 3y^2x)dy = 0.$$

Or

10. (a) Define and give an example of variables separable equation.

(b) Solve $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$.

(104MA24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Mathematics

TOPOLOGY

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

(5 × 14 = 70)

UNIT I

1. (a) Define a metric space. Give two examples that are not subsets of R^n.

 (b) Show that a continuous function $f:(X,d)\to(Y,\rho)$ maps convergent sequences to convergent sequences.

Or

2. (a) Prove that the open ball $B(x,r)$ is an open set in a metric space.

 (b) Show that R with the usual metric is a complete space.

UNIT II

3. (a) Define a topological space. Give two non-metric examples.

 (b) Prove that any union of open sets is open in a topological space.

Or

4. (a) State the axioms that define a topology.

 (b) Let $X = \{a,b,c\}$. List all topologies on X.

UNIT III

5. (a) State and explain the Ascoli-Arzelà theorem.

 (b) State and prove Tychonoff's theorem for finite products.

Or

6. (a) Give an example of a compact space which is not sequentially compact.

   (b) Prove that every compact metric space is complete and totally bounded.

UNIT IV

7. (a) Define T_0, T_1 and T_2 (Hausdorff) spaces with examples.

   (b) Prove the Tietze Extension Theorem for normal spaces.

Or

8. (a) Give an example of a space that is T_1 but not T_2.

   (b) Prove that the real line R with usual topology is normal.

UNIT V

9. (a) Define connectedness and show that the image of a connected space under a continuous map is connected.

   (b) State and prove the Urysohn Embedding Theorem.

Or

10. (a) Prove that the continuous image of a connected space is connected.

    (b) Let X be a compact Hausdorff space. Show how it can be embedded in a cube.

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Mathematics

ADVANCED DISCRETE MATHEMATICS

Time : Three hours Maximum : 70 marks

Answer ONE question From Each Unit.

(5 × 14 = 70)

All questions carry equal marks.

UNIT I

1. (a) Define tautology, contradiction and contingency with examples.

   (b) Construct a truth table and determine whether the following formula is a tautology $(P \to Q) \vee (\neg P)$.

Or

2. (a) Explain the concept of duality law in prepositional logic.

   (b) Reduce the statement

   $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ and verify whether it is a tautology.

UNIT II

3. (a) What is consistency of premises? Explain with an example using the indirect method of proof.

   (b) Convert the following into prenex normal form and identify free and bound variables:

   $\forall x (P(x) \to \exists y\, Q(y, x))$.

Or

4. (a) State and explain rules of inference in predicate logic.

   (b) Using rules of inference, derive the conclusion :

   Given :

   (i) $\forall x (P(x) \to Q(x)$

   (ii) $P(a)$

   Prove $Q(a)$.

UNIT III

5. (a) Define a finite state machine. Explain with an example how to construct a state table and diagram.

   (b) Given a state table for a machine with three states A, B, C and input symbols {0, 1}, analyze the behavior and identify reachable states from A.

Or

6. (a) Differentiate between Mealy and Moore machines with suitable diagrams.

   (b) Construct the state diagram for a machine that accepts binary strings ending with '01'.

UNIT IV

7. (a) Define a distributive lattice. Prove that every Boolean algebra is a distributive lattice.

   (b) Let L={1,2,3,6} under divisibility. Determine whether LLL forms a lattice. Draw its Hasse diagram.

Or

8. (a) Prove or disprove: Every lattice is a Boolean algebra.

   (b) Express the Boolean polynomial

   $F(x, y, z) = xy + x'z + yz$ in minimal form.

UNIT V

9. (a) Define ideals and filters in lattices. Give examples.

   (b) Construct a switching circuit for the Boolean expression $F = AB + A'C$ and simplify it using Boolean algebra.

Or

10. (a) Explain the concept of minimal forms of Boolean expressions using Karnaugh maps.

   (b) Simplify the Boolean function F(A,B,C)= Σ(1,3,5,7) using a Karnaugh map and draw the resulting switching circuit.