M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Statistics

PROBABILITY THEORY AND DISTRIBUTIONS

Time : Three hours Maximum : 70 marks

Answer ALL questions from the following.

$(5 \times 14 = 70)$

UNIT I

1. (a) Define limit supremum and limit infimum of sequence of sets. Prove that Lim inf $A_n \subseteq Lim\sup A_n$.

   (b) Prove that the probability measure is countably additive. (14)

Or

2. (a) Define field and σ-field. Give examples.

   (b) Define statistical independence and prove that if $A \subset B$ and $P(B) > 0$, then $P(A \mid B) \leq 1$. (14)

UNIT II

3. (a) Define measurable function. Show that a composition of measurable functions is measurable.

   (b) Define and explain Holder's inequality. (14)

Or

4. (a) Derive the inversion formula for the characteristic function.

   (b) Define marginal and conditional distributions Give one example. (14)

UNIT III

5. (a) Prove Kintchine's W.L.L.N for i.i.d. random variables with finite mean.

   (b) Give an example to show that convergence in probability does not imply almost sure convergence. (14)

Or

6. (a) Explain the concepts of convergence in distribution and almost sure convergence.

   (b) Prove that convergence in mean square implies convergence in probability. (14)

UNIT IV

7. (a) Define multinomial distribution. Derive the expected value and variance of its components.

   (b) Define and explain truncated Poisson distribution. (14)

Or

8. (a) Derive the moment generating function of Laplace distribution.

   (b) Write the applications and properties of the logistic distribution. (14)

UNIT V

9. (a) Derive the distribution function of the r-th order statistic.

   (b) Obtain the joint p.d.f. of order statistics in the case of continuous distribution. (14)

Or

10. (a) Define range and derive its distribution for exponential distribution.

    (b) Give two applications of order statistics in reliability theory. (14)

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(102ST24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025

First Semester

Statistics

STATISTICAL COMPUTING USING R

Time : Three hours Maximum : 70 marks

Answer ALL questions from the following.

(5 × 14 = 70)

UNIT I

1. (a) Discuss the different operators in R (arithmetic, relational, logical, assignment). Write code examples.

   (b) Explain assignment, relation and miscellaneous operators using in 'R'. (14)

Or

2. (a) What are factors in R? How are factor variables created and manipulated? Explain with gl( ) and other functions.

   (b) Describe integer, raw, numeric, logical, complex. Data types in 'R'. (14)

UNIT II

3. (a) Explain the creation and operations on matrices and lists in R with examples.

   (b) Explain about writing data files from other files. (14)

Or

4. (a) How can data be read into and saved from R?

   (b) Explain the use of read.csv( ), write.table( ), save( ), load ( ) and file.choose ( ) functions.

(14)

UNIT III

5. (a) Explain the different types of loops in R with examples.

   (b) Write an R program using for loop to compute factorial of a number. (14)

Or

6. (a) How are functions defined and called in R?

(b) Write a function to compute correlation and another to perform a paired t-test. (14)

UNIT IV

7. (a) Describe the built-in R functions for normality tests (Shapiro-Wilk, Kolmogorov-Smirnov). Write code to test normality for a given dataset.

(b) Explain Wilcoxon Mann-Whitney U-test by using R. (14)

Or

8. (a) Describe R-Codes for Exponential.

(b) You are provided with the following data representing the number of successes observed in a series of experiments, where each experiment consists of 4 independent Bernoulli trials (i.e., n=4).

Number of Successes (x)	Observed Frequency
0	5
1	12
2	18
3	10
4	5

Task:

(i) Fit a binomial distribution to the given data by estimating the probability of success ppp from the data.

(ii) Write R code to:
- Estimate the value of p
- Compute the expected frequencies for each number of successes
- Perform a chi-square goodness-of-fit test

(iii) Clearly report the chi-square test statistic, degrees of freedom, and p-value.

(iv) Based on the p-value, comment on the goodness of fit of the binomial model to the observed data. (14)

UNIT V

9. (a) Write-R code to create pie charts, bar charts, and Q-Q plots.

(b) How can you control and modify plot elements like axes, text, grid, and legend?

(14)

Or

10. (a) Explain Built in R-syntax for one-way ANOVA and two-way ANOVA.

(b) Write about box Whisker plots, dot plot Scatter plots in R. (14)

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(103ST24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Statistics

ESTIMATION

Time : Three hours Maximum : 70 marks

Answer ALL questions from the following.

(5 × 14 = 70)

UNIT I

1. (a) Define and explain the concepts of parameter, parametric space, statistic, estimator and standard error with suitable examples.

   (b) State and prove the Fisher-Neyman Factorization Theorem. (14)

Or

2. (a) Explain the concept of sufficiency with examples. What is the difference between joint density and likelihood function?

   (b) Define complete and minimal sufficiency. Give examples to illustrate. (14)

UNIT II

3. (a) Define UMVUE. Explain the properties and regularity conditions for UMVUE.

   (b) State and prove the Rao-Blackwell Theorem. (14)

Or

4. (a) What is the Cramer-Rao inequality? State the condition for equality and provide an example.

   (b) Define consistency and efficiency of an estimator. Explain CAN and CAUN estimators. (14)

UNIT III

5. (a) Derive the MLE for the parameter of exponential distribution.

   (b) Explain the method of moments and compare it with MLE. (14)

Or

6. (a) Explain the concept of percentile estimation with an example.

(b) Define minimum chi-square and modified minimum chi-square methods. (14)

UNIT IV

7. (a) What is interval estimation? How are confidence intervals constructed using pivots?

(b) Explain the relationship between confidence intervals and hypothesis testing. (14)

Or

8. (a) Define prior and posterior distributions. Explain the concept of Bayes Estimator with an example.

(b) What is a loss function and risk function? Define and discuss Minimax Estimator. (14)

UNIT V

9. (a) Define Type I and Type II censoring in the context of lifetime data analysis.

(b) The lifetimes (in hours) of 6 electronic components were tested in a life test experiment. The experiment was terminated at a pre-fixed time T = 10 hours (Type I censoring). The following failure times (in hours) were observed:

2.3, 4.1, 6.7, 9.5

The remaining 2 units did not fail before time 10 and are therefore right-censored at 10 hours.

Tasks:

Define Type I and Type II censoring clearly.

Assume that the lifetimes follow an exponential distribution with probability density function:

$F(t;\lambda) = \lambda e^{-\lambda t},\ t \geq 0$

Obtain the maximum likelihood estimate (MLE) of the parameter λ using the given censored data. (14)

Or

10. (a) Discuss truncated distributions and their application in estimation.

(b) Explain the procedure of constructing confidence intervals using pivots for censored normal data. (14)

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(104ST24)

M.Sc. DEGREE EXAMINATION, JUNE/JULY 2025.

First Semester

Statistics

SAMPLING THEORY

Time : Three hours Maximum : 70 marks

Answer ALL questions from the following.

(5 × 14 = 70)

UNIT I

1. (a) Describe the method of stratified sampling. Derive the expression for its variance.

   (b) Explain gain in precision due to stratification with an example. (14)

Or

2. (a) Discuss systematic sampling for populations with linear trend.

   (b) Explain Yates end correction in systematic sampling. (14)

UNIT II

3. (a) Define PPS sampling. Explain the method of selection and estimation with replacement.

   (b) Describe Murthy's estimator for a sample of size two. (14)

Or

4. (a) Explain Horvitz-Thompson estimator and derive its variance.

   (b) Discuss Midzuno-Sen sampling scheme. (14)

UNIT III

5. (a) Explain two-stage sampling with equal number of second-stage units and derive the estimate of population mean.

   (b) Derive the expression for variance and its estimation in two-stage sampling. (14)

Or

6. (a) Define double sampling for stratification. Derive the expression for variance of the estimated mean.

(b) Explain optimum allocation in double sampling. (14)

UNIT IV

7. (a) Discuss double sampling for regression estimation and obtain the regression estimator.

(b) Explain sources and types of non-sampling errors. (14)

Or

8. (a) Describe the Hansen and Hurwitz technique for non-response adjustment.

(b) Explain Deming's model for non-sampling errors. (14)

UNIT V

9. (a) Define ratio estimator. Derive its bias and mean square error.

(b) Compare ratio estimator with mean per unit estimator. (14)

Or

10. (a) Explain difference estimator and derive its variance.

(b) Compare regression estimator with ratio and mean per unit estimators. (14)

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