(DM 01)

ASSIGNMENT-1
M.Sc. DEGREE EXAMINATION, JUNE 2022.
First Year
Mathematics
ALGEBRA
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) If G is a finite abelian group of order n and m is positive integer prime to n¸ then show that the mapping $\sigma : x \to x^m$ is an automorphism of G.

   (b) State and prove Sylow's theorem for abelian groups.

2. (a) Define a composition series of a finite group. Prove that any two composition series of a finite group are equivalent.

   (b) Show that conjugacy is an equivalence relation on G.

3. (a) If R is a commutative ring with unity in which each ideal is prime, then prove that R is a field.

   (b) Describe all finite abelian groups of order $2^4 3^4$.

4. (a) Find the non-trivial ideals of the ring $P = \begin{bmatrix} Z & Q \\ 0 & 0 \end{bmatrix}$.

   (b) Show that a finite integral domain is a field.

5. (a) If $p(x)$ is a polynomial in $F(x)$ of degree $n \geq 1$ and irreducible over F then show that there is an extension E of F such that $[E : F] = n$ in which $P(x)$ has a root.

   (b) What is Euclidean ring? Explain a particular Euclidean ring.

(DM 01)

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

Mathematics

ALGEBRA

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. Show that every integral domain can be imbedded in a field.

2. (a) State and prove the division algorithm for polynomial rings over a commutative integral domain.

   (b) Show that the polynomial $f(x)\ \ E F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common root.

3. (a) Show that a group *G* insolvable if and only if $G^{(k)} = e$ for some integer *k*.

   (b) Show that the general polynomial $p(x) = x^n + a_1 x^n + ... + a_n$ for $n \geq 5$ is not solvable by radicals.

4. (a) Prove that any totally ordered set is a distributive lattice.

   (b) Show that a lattice of invariant sub groups of any group is modular.

5. (a) Prove that a partially ordered set with a least element O such that every non-empty subset has a least upper bound is a complete lattice.

   (b) Prove that the complement Q' of any element *a of* a, Boolean algebra *B* is uniquely determined and also, prove that $(a \vee b)' - a' \wedge b';\ (a \wedge b)' = a' \vee b'$ in *B*.

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

Mathematics

ANALYSIS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Let $\{E_n\}, n=1,2,3.....,$ be the sequence of countable sets and $S=\bigcup_{n=1}^{\infty} E_n$. Then '$S$' is countable.

   (b) Compact subsets of Metric spaces are closed.

2. (a) Prove that every k-cell is compact.

   (b) If p is a limit point of a set E, then every neighbourhood of p contains infinitely main points of E.

3. (a) Let $\{P_n\}$ be a subsequence in a Metric space X

   (i) $\{P_n\}$ converges to $P \in X$ if and only if every neighbourhood of p contains P_n for all but finitely many 'n'.

   (ii) If $p \in X, p' \in X$ and if $\{P_n\}$ converges to P and P' then $P = P'$.

   (iii) If $\{P_n\}$ converges then $\{P_n\}$ is bounded

   (b) If $\sum a_n$ converges and if $\{b_n\}$ is monotonic and bounded, prove that $\sum a_n b_n$ converges.

4. (a) Let '*f*' be a real uniformly continuous function on the bounded set *E* in R'. Prove that *f* is bounded on *E*.

(b) Suppose *f* is a continuous mapping of a compact metric space *X* into a metric space *Y*. Then $f(X)$ is compact.

5. (a) Let $f \in R(\alpha)$ on $[a, b]$ $\Leftrightarrow$ for every $\in > 0$ there exists a partition '*p*' such that

$$U(p, f, \alpha) - L(p, f, \alpha) < \in$$

(b) If *f* maps $[a, b]$ in R^k and if $f \in R(\alpha)$ for some monotonically increasing function 'α' on $[a, b]$ then $|f| \in R(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha$.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

Mathematics

ANALYSIS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Suppose $f \geq 0$ is continuous on $[a, b]$ and $\int_a^b f(x)\, dx = 0$, prove that $f(x) = 0 \;\forall\; x \in [a, b]$.

(b) Suppose F and G are differentiable functions on $[a, b]$, $F' = f \in R$ and $G' = g \in R$ then

$$\int_a^b F(x)\; G(x)\, dx = F(b)\; G(b) - F(a)\; G(a) - \int_a^b f(n)\; G(n)\, dx$$

2. (a) The sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\in > 0$, ∃ an integer N such that $M \geq N,\, n \geq N,\, x \in E$

$$\Rightarrow \left| f_n(x) - f_m(x) \right| \leq \in$$

(b) State and prove Weierstrass approximation theorem.

3. (a) If K is a compact metric space, if $f_n \in \mathscr{C}\,(K)$ for $k = 1, 2, 3...$ and if $\{f_n\}$ converges uniformly on k, then $\{f_n]$ is equicontinuous on k.

(b) Let 'α' be monotonically increasing on $[a, b]$. Suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1, 2, 3,...$ and suppose $f_n \to f$ uniformly on $[a, b]$, then $f \in R(\alpha)$ and $\int_a^b f dx = \lim_{n\to\infty} \int_a^b f_n dx$.

4. (a) State and prove Lebesgue's dominated Converges theorem.

(b) Suppose f is measurable and non negative on X, for $A \in \mathfrak{R}$ define $\phi(A) = \int_A f \, d\mu$ then ϕ is countably additive on $\mathfrak{R}$.

5. (a) State and prove Fatou's theorem.

(b) Let $\{f_n\}$ be a sequence of measurable functions. For $x \in X$

$g(x) = \sup f_n(x) \qquad n = 1, 2, 3, \ldots$ Then g and h are measurable.

$h(x) = \lim_{n \to \infty} \sup f_n(x)$

(DM 03)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

COMPLEX ANALYSIS AND SPE. FUNCTIONS AND PARTIAL DIF. EQU.

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

SECTION — A

1. (a) Find a power series solution of the Legendre's equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$.

   (b) State and prove Rodrigque's formula for Legendre's equation.

2. (a) Show that for any function $f(x)$, for which the n^{th} derivative is continuous $\int_{-1}^{1} f(x) P_n(x) dx = \frac{1}{2^n n!} \int_{-1}^{1} (1-x^2)^n f^n(x) dx$.

   (b) We shall prove that

$$\int_{-1}^{1} P_m(x) P_n(x) dx = \begin{cases} 0, & m \neq n \\ \frac{2}{2n+1}, & m = n \end{cases}.$$

3. (a) Prove that

$$\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\, dx = \begin{cases} 0, & \alpha \neq \beta \\ \frac{1}{2}[J_n + (\alpha)]^2, & \alpha = \beta \end{cases}$$

Where α, β are the roots of $J_n(x) = 0$.

(b) Prove that

$$\frac{d}{dx}\{x J_n(x) J_{n+1}(x)\} = J_n^2(x) - J_{n+1}^2(x).$$

4. (a) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

(b) Find the general solutions of $(D^2 - DD' + D' - 1)\ Z = \cos(x+2y)$.

5. (a) Solve $(D^2 + DD' - 6\ D'^2)z = \cos(2x+y)$.

(b) Solve $(D^2 - D')z = 2y - x^2$.

(DM 03)

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

COMPLEX ANALYSIS AND SPE. FUNCTIONS AND PARTIAL DIF. EQU.

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Use De Moivre's theorem to solve the equation $x^5 + 1 = 0$.

   (b) Prove that if G is open and connected and $f : G \to C$ is differentiable with $f'(z) = 0$ for all z in G, then f is constant.

2. (a) State and prove the fundamental theorem of algebra.

   (b) Let G be an open set and let $f : G \to C$ be a differentiable function. Then prove that f is analytic on G.

3. (a) Find the Laurent's expansion of $f(z) = \dfrac{7z-2}{z(z+1)(z-2)}$ in the region $1 < z+1 < 3$.

   (b) State and prove the homotopic version of Cauchy's theorem.

4. (a) State and prove open mapping theorem.

   (b) Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the region.

(i) $|z| < 1$,

(ii) $1 < |z| < 2$.

5. (a) By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$.

(b) Show that $\int_0^{\infty} \frac{\sin x}{x} \, dx = \frac{\pi}{2}$.

(DM 04)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Let $\phi_1, \phi_2, \phi_3 \ldots \phi_n$ be the n solution of $L(y)=0$ on I satisfying $\phi_i^{(i-1)}(x_0)=1$. $\phi_i^{(J-1)}(x_0)=0$ for $i \neq j$. If ϕ is any solution of $L(y)=0$ on I, there are n constants $c_1, c_2, c_3 \ldots\ldots\ldots c_n$ such that

   $\phi = C_1\,\phi_1 + C_2\,\phi_2 + C_3\,\phi_3 \ldots\ldots + C_n\,\phi_n$.

   (b) Consider the equation

   $L(y) = y'' + a_1(x)\,y' + a_2(x)\,y = 0$, where a_1, a_2 are continuous on same interval I. Let $\phi_1\,\phi_2$ and ψ_1, ψ_2 be two bases for the solutions of $L(y)=0$. Show that there is a non zero constant $'k'$ such that $W(\psi_1, \psi_2)(x) = kW(\phi_1, \phi_2)(x)$.

2. (a) One solution of $x^2\,y''' - 3x^2\,y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solution $x > 0$

   (b) Find all solutions of the equation $y'' - \dfrac{2}{x^2}\,y = x \quad 0 < x < \infty$.

3. (a) Find a real valued solution of $y' = \frac{e^{x-y}}{1+e^{x}}$.

(b) Prove that, the necessary and sufficient conditions for the equation

$M(x, y)dx + N(x, y)\, dy = 0$ is exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

4. (a) (i) Show that the function *f* is given by

$f(x, y) = x^2|y|$ satisfies Lipschitz condition on R : $|x| \le 1, (y) \le 1$

(ii) Show that $\frac{\partial f}{\gamma y}$ does not exist $< t(x, 0)$ if $x \ne 0$.

(b) Show that every initial value problem $y' = f(x, y),\ y(0) = y_0,\ (|y_0| < \infty)$ has a solution which exists for $|x| < 1$.

5. (a) Solve $y\,y'' + 4(y')^2 = 0$

(b) Give an example of a system of differential equations which arise in the study of dynamics of central forces and planetary motion.

(DM 04)

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, JUNE 2022.

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1(a) Find a solution ϕ of the system

$y'_1 = y_1,\ y'_2 = y_1 + y_2$ which satisfies $\phi(0) = (1, 2)$.

(b) Find a solution ϕ of $y'' = 1 + (y')^2$ satisfying $\phi(0) = 1$, $\phi(0) = -1$.

2 (a) Show that

$$G(x, t) = \frac{1}{\sin h1}\begin{cases} \sin h(t-1)\sin h\, x & 0 \le x \le t \\ \sin h(-\sin h\,(x-1) & t \le x \le 1 \end{cases}$$

is the green function of the problem $y'' - y = 0, y(0) = 0 \quad y(1) = 0$. Hence solve the problem

$y'' - y = 2\sin x,\ y(0) = 0\ y(1) = 2$.

(b) Find the general solution of $y''-3y'+2y=f(x)$, $-\infty<x<\infty$ where f is a continuous function and then evaluate the general solution when $f(x)=x$.

4(a) Show that if z_0, z_1, z_2, z_3 are any four different solutions of the Riccati equation

$z^1+a(x)z+b(x)z^2+c(x)=0$ then Show that

$\frac{z-z_2}{z-z_1}=\frac{z_3-z_1}{z_3-z_2}$ = constant.

(b) Suppose a particle moves on a circle through origin and is acted on by a central force $F(r)$. Show that $F(r)$ is proportional to r^{-5}.

5(a) State and prove strum's comparison theorem.

(b) Discuss the oscillations of the Bessel equation. $x^2y''-xy'+\left(x^2-n^2\right)y=0$ where n is a constant.

6(a) State and prove Picone's Identity

(b) State and prove Abel's formula.
