DM01

ASSIGNMENT – 1

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020

(First Year)

MATHEMATICS

Algebra

MAXIMUM MARKS – 30

ANSWER ALL QUESTIONS

Q1) a) If ϕ is a homomorphism of G into $\overline{G}$ with kernel ***K***, then prove that ***K*** is a normal sub group of G.

b) If G is abelian or order O(G) and P^{α}/ O(G), $P^{\alpha+1}$ O(G), then prove that there is a unique subgroup of G of order P^{α}.

Q2) a) If G is a group, then show that A (G), the set of automorphisms of G, s also a group.

b) State and prove Cayley's theorem.

Q3) a) If p is prime number and p^{α}/O(G), then prove that G has a subgroup of order p^{α}.

b) Let G be a group and suppose that G is the internal direct product of $N_1,\ldots,N_n$. Let T = $N_1\ldots N_n$. Then prove that G and T are isomorphic.

Q4) a) Prove that a finite integral domain is a field.

b) If ϕ is a homomorphism of R into R', then prove that (1) $\phi(0) = 0$ (2) $\phi(-a) = -\phi(a)$ for every $a \in R$.

Q5) a) If R is unique factorization domain, then show that R[x] is also unique factorization domain.

b) If R is an integral domain with unit element, prove that any unit in R[x] must already be a unit in R.

DM01

ASSIGNMENT – 2

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020

(First Year)

MATHEMATICS

Algebra

MAXIMUM MARKS – 30

ANSWER ALL QUESTIONS

Q1) a) Prove that the element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.

b) If $a \in K$ is a algebraic of degree n over F, then prove that [F(a) : F] = n.

Q2) a) Prove that if α, β are constructible, then so are $\alpha \pm \beta\ \alpha\ \beta$ and α / β (when $\beta \neq 0$)

b) Prove that a circle in Fhas an equation of the form $x^2 + y^2 + ax + by + c = 0$, with a, b, c in F.

Q3) a) Show that the polynomial $p(x) = x^3 - 3x - 3$ over Q are irreducible and have exactly two non-real roots.

b) In S_5, show that (1 2) and (1 2 3 4 5) generate S_5.

Q4) a) Show that the partially ordered set of subgroups of a cyclic group of prime power is a chain.

b) Show that any complete Lattice has a zero and an element.

Q5) a) Prove that the following two types of abstract systems are equivalent.

(1) Boolean algebra

(2) Boolean ring with identity

b) Prove that any ring for which there exist a prime p such that $pa = 0$, $a^p = a$ for every a in the ring is commutative.

DM02

ASSIGNMENT – 1

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020

(First Year)

MATHEMATICS

Analysis

MAXIMUM MARKS – 30

ANSWER ALL QUESTIONS

Q1) **a)** Prove that every infinite subset of a countable set A is countable.

b) Let $\{E_n\}$, $n = 1,2,3, \ldots$, be a sequence of countable sets, and put , then prove that $S = \bigcup_{n=1}^{\infty} E_n$, is countable.

Q2) **a)** Prove that every bounded infinite subset of R^k has a limit point in R^k.

b) Prove that every k - *cell* is compact.

Q3) **a)** The sub sequential limits of a sequence $\{p_n\}$in a metric space X form a closed subset of X.

b) Suppose $\{s_n\}$is monotonic. Then prove that $\{s_n\}$ converges if and only if it is bounded.

Q4) **a)** If f is continues mapping of a compact metric space X into R^k, then prove that $f(X)$ is closed and bounded.

b) If f is continuous mapping of a compact metric space X into Y. Then f is uniformly continuous on X.

Q5) **a)** If γ' is continuous on $[a, b]$, then prove that γ' is rectifiable, and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$.

b) If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, prove that $f \notin \mathscr{R}$ on $[a, b]$ for any $a < b$.

DM02

ASSIGNMENT – 2

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020

(First Year)

MATHEMATICS

Analysis

MAXIMUM MARKS – 30

ANSWER ALL QUESTIONS

Q1) **a)** Prove that $f \in \mathscr{R}$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exist a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$.

b) If P^* is a refinement of P, then prove that $L(p,f,\alpha) \leq L(P^*,f,\alpha)$.

Q2)

a) Prove that the sequence of function $\{f_n\}$ defined on E, converges uniformly on E if and if only for every $\varepsilon > 0$ there exist an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $|f_n(x) - f_m(x)| \leq \varepsilon$.

b) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n$ $(x \in E, n = 1,2,3...)$ then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

Q3)

a) If K is a compact metric space, if $f_n \in C(k)$ for $n = 1,2,3,...$ and if $\{f_n\}$ is pointwist bounded and equicontinuous on K, then prove that (1) $\{f_n\}$ is uniformly bounded on K(2) $\{f_n\}$ is contains a uniformly convergent subsequence.

b) If K is a compact metric space, if $f_n \in C(k)$ for $n = 1,2,3,...$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

Q4)

a) If f is measurable, then prove that $|f|$ is measurable.

b) Let f and g are measurable real-valued functions defined on X, let F be real and continuous on R^2, and put $h(x) = F(f(x), g(x))$, $(x \in X)$ then prove that h is measurable.

Q5)

a) If $f \in \mathcal{L}(\mu)$ on E, then prove that $|f| \in \mathcal{L}(\mu)$ on E and $\left|\int_E f d\mu\right| \leq \int_E |f| d\mu$.

b) Suppose that $f = f_1 + f_2$, where $f_i \in \mathcal{L}$ (μ) on E$(i = 1,2,3........)$, then prove that $f \in \mathcal{L}(\mu)$ and

$$\int_E f d\mu = \int_E f_1 d\mu + = \int_E f_2 d\mu.$$

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DM03

ASSIGNMENT – 1
M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020
(First Year)
MATHEMATICS
Complex Analysis and Special Functions and Partial Differential Equations
MAXIMUM MARKS – 30
ANSWER ALL QUESTIONS

Q1)

a) Prove that

$$\left(1-2xz+z^2\right)^{1/2}=\sum_{n=0}^{\infty} z^n p_n(x),\ |x|\le 1,\ |z|<1.$$

b) If $u_n = \int_{-1}^{1} x^{-1} p_n(x)\ p_{n-1}(x)\ dx$, then show that $nu_n + (n-1)u_{n-1} = 2$, hence evaluate u_n.

Q2)

a) Show that

i) $\int_0^x x^{-n} J_{n+1}(x)\ dx = \frac{1}{2^n \Gamma(n+1)},\ n>1.$

ii) $\int_0^\infty x^{-n} J_{n+1}(x)\ dx = \frac{1}{2^n \Gamma(n+1)},\ n>-\frac{1}{2}.$

b) Show that

i) $\int_0^x x^3 J_0(x)dx = x^3 J_1(x) - 2x^2 J_2(x)$

ii) $\int_0^1 x^3 J_0(x)dx = 2J_0(1) - 3J_1(1)$

Q3) a) Solve $(yz+2x)dx + (zx-2z)dy + (xy-2y)dz = 0$.

b) Solve
$z(y+z)dx + z(t-x)dy + y(x-t)dz + y(y+z)dt = 0$.

Q4) a) Solve $(x^2-y^2-yz)p + (x^2-y^2-zx)q = z(x-y)$.

b) $\left(D^2+2DD'+D'^2\right)z = 2\cos y - x\sin y$.

Q5) a) Solve $(D+D'-1)(D+2D'-3)z = 4+3x+6y$.

b) Solve $(r-t)xy - s(x^2-y^2) = qx - py$ by Monge's method.

DM03

ASSIGNMENT – 2
M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020
(First Year)
MATHEMATICS
Complex Analysis and Special Functions and Partial Differential Equations
MAXIMUM MARKS – 30
ANSWER ALL QUESTIONS

Q1)

a) Prove that $||z|-|w|| \leq |z-w|$ and give necessary and sufficient conditions for equality.

b) Let $Z = \text{cis}\,\frac{2\pi}{n}$ for an integer $n \geq 2$. Show that $1 + z + \ldots + z^{n-1} = 0$.

Q2)

a) If $\sum_{n=0}^{\infty} a_n (z-a)^n$ is a given power series with radius of convergence R, then prove that $R = \lim |a_n / a_{n+1}|$ if this exist.

b) Let G be either the whole plane C or some open disk. If $u : G \to R$ is a harmonic function, then show that u harmonic conjugate.

Q3) a) State and prove Cauchy's integral formula in second version.

b) Evaluate $\int_{\gamma} \frac{dz}{z^2+1}$ where $\gamma(\theta) = 2|\cos 2\theta| e^{i\theta}$ for $0 \leq \theta \leq 2\pi$.

Q4) a) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$.

b) State and prove general version of Rouche's theorem for curves other than circle in G.

Q5) a) State and prove Maximum Modulus theorem.

b) Evaluate $\int_0^{\infty} \frac{x^2 dx}{x^4 + x^2 + 1}$.

DM04

ASSIGNMENT – 1
M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020
(First Year)
MATHEMATICS
Theory of Ordinary Differential Equations
MAXIMUM MARKS – 30
ANSWER ALL QUESTIONS

Q1)

a) State and prove existence theorem.

b) Let $\phi_1, \ldots \phi_n$ be n solutions of $L(y) = 0$ on an interval I, and let x_0 be any point in I. Then prove that $W(\phi_1, \ldots \phi_n)(x) = \left| \exp\left[-\int_{x_0}^{x} a_1(t)\,dt\right]\right| W(\phi_1, \ldots \phi_{1n})(x)$

Q2)

a) Verify the function ϕ_1 satisfies the equation $x^2 y'' - 7xy' + 15xy = 0$, $\phi_1(x) = x^2$, $(x > 0)$ and find a second independent solution.

b) State and prove existence theorem for Analytic coefficients.

Q3)

a) Find all real - valued solutions of $y' = \dfrac{e^{x-y}}{1+e^x}$.

b) Let M, N be two real valued functions which has continuous first partial derivatives on some rectangle R : $|x - x_0| \le a, |y - y_0| \le b$. Then show that $M(x,y) + N(x,y)\,y' = 0$ is exact in R if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

Q4)

a) Show that a function ϕ is a solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$.

b) Find the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for $y' = x^2 + y^2$, $y(0) = 0$.

Q5)

a) Find the solution $\varphi(x)$ of $y'' = 1 + (y')^2$ which satisfies $\varphi(0) = 0$, $\varphi'(0) = 0$.

b) Suppose that f is a continuous function on an interval $|x - x_0| \leq a$. Show that the solution φ of the initial value problem $y'' = f(x)$, $y(x_0) = \alpha$, $y'(x_0) = \beta$ can be written as

$$\varphi(x) = \alpha + \beta\,(x - x_0) + \int_{x_0}^{x} |x - t| f(t)dt.$$

DM04

ASSIGNMENT – 2

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2020

(First Year)

MATHEMATICS

Theory of Ordinary Differential Equations
MAXIMUM MARKS – 30
ANSWER ALL QUESTIONS

Q1)

a) State and prove local existence theorem.

b) Consider system where $y_1' = ay_1 + by_2$, $y_2' = -by_1 + ay_2$ a, b are real constants,

(1) If $\phi = (\phi_1, \phi_2)$ is any solution with values in R_2, show that $\|\phi(x)\| = \|\phi(0)\| e^{ax}$, where $\|\phi(x)\| = [\phi_1^2(x) + \phi_1^2(x)]^{1/2}$.(2) Verify that the solution satisfies $\phi(0) = (1,0)$ is given by

$\phi(x) = e^{ax}(\cos bx, -\sin bx)$.

Q2)

a) By reduction to linear equation solve the Riccati's equation $y' = -2y^2 - 5y - 2$.

b) Find the greens function of the boundary value problem $y'' = -f(x), y(0) = 0, y(1) = 0$.

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Q3)

a) Show that if z_1, z_2, z_3 are any four different solutions of the Riccati equation $y' + a(x)y + b(x)y^2 + c(x) = 0$, then show that $\frac{y - y_2}{y - y_1} = \frac{y_3 - y_1}{y_3 - y_2}$.

b) Find the functions $z(x)$, $k(x)$, $m(x)$ such that $z(x)\left[x^2 y'' - 2xy' + 2y\right] = \frac{d}{dx}\left(k(x)\, y' + m(x) y\right)$ and hence solve $x^2 y''$ - $2xy' + 2y = 0, x > 0$.

Q4)

a) State and prove strum seoaration theorem.

b) Solve $x^2 y''$ - $2xy' + (2 + x^2)y = 0, x > 0$.

Q5)

a) State and prove Gronwall's inequality.

b) Discuss the oscilation of Bessel equation $x^2 y'' - xy' + (x^2 - n^2)y = 0$.