(101MA24)

M.Sc DEGREE EXAMINATIONS, Model QP
Mathematics - First Semester
ALGEBRA

Time : Three hours **Maximum : 70 marks**

Answer ONE question from each Unit. **(5 x 14 = 70)**

UNIT - I

1. (a) State and prove Cauchy's theorem for abelian groups.
   (b) If H and K are finite subgroups of a group G of orders O(H) and O(K) respectively, then prove that O(HK) = $\frac{o(H)o(K)}{o(H\cap K)}$

OR

2. (a) Prove that I(G) $\approx$G/Z, where I(G) is the group of inner automorphisms of the group G and Z is the center of G.
   (b) If G is a group then prove that A(G), the set of automorphisms of G is also a group.

UNIT – II

3. (a) State and prove Caylay's theorem.
   (b) Prove that If G is a finite group of order p^2, where p is a prime number then G is abelian.

OR

4. (a) If p is a prime number, G is a finite group and p^n / O(G) then prove that G has a subgroup of order p^n.
   (b) State and third part of the Sylow's theorem.

UNIT – III

5. Prove that every finite abelian group is the direct product of cyclic groups.

OR

6. (a) Prove that a finite integral domain is a field.
   (b) If U is an ideal of the ring R, then prove that R/U is a ring and is a homeomorphic image of R.

UNIT – IV

7. (a) If R is a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
   (b) Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists an element $a_0 \in A$ such that A consists exactly of all a_0 x as x ranges over R.

OR

8. (a) Let R be a Euclidean ring. Then prove that any two elements a and b in R have a greatest common divisor d and d = xa + yb for some x, y $\in$ R.
   (b) State and prove Eisenstein Criterian.

UNIT – V

9. (a) If R is a unique factorization domain and if p(x) is a primitive polynomial in R[x], then prove that it can be factored in a unique way as the product of irreducible elements in R[x].
   (b) Prove that Hom(V,W) is a vector space over the field F by defining addition and scalar multiplication.

OR

10. (a) If V is the internal direct sum of $U_1,..,,U_n$ then prove that V is isomorphic to the external direct sum of U_1 ,.. ., U_n .
    (b) Prove that if V and W are finite dimensional vector spaces of dimensions m and n respectively over F then Hom (V, W) is of dimension mn over F.

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M.Sc DEGREE EXAMINATIONS, Model QP
Mathematics - First Semester
ANALYSIS-1

Time : Three hours **Maximum : 70 marks**

Answer **ONE** question from Each Unit. (5 x 14 = 70)

UNIT-I

1. A) For any two real sequences $\{a_n\}, \{b_n\}$ prove that
   $\lim_{n\to\infty} sup\,(a_n + b_n) \leq \lim_{n\to\infty} sup\, a_n + \lim_{n\to\infty} sup\, b_n$
   provided the sum on the right is not of the form $\infty - \infty$.
   B) State and Prove Root test.

(OR)

2. A) Let $a_n \neq 0$ then $\sum a_n$
   (i) Converges if $\lim sup \left[\left|\frac{(a_n+1)}{a_n}\right|\right] < 1$ and
   (ii) Diverges if $\lim \left[\left|\frac{(a_n+1)}{a_n}\right|\right] > 1$.
   B) State and Prove Mertens Theorem.

UNIT-II

3. A) **(a) :** Let f_1, f_2, f_3, f_4 be real functions on a metric space X, and let f be the mapping of X into $\mathbb{R}^k$ defined by $f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$ $(\forall x \in X)$; then f is continuous if and only if each of the functions $f_1, f_2, \ldots\ldots f_k$ are continuous.
   (b): If f and g are continuous mappings of X into $\mathbb{R}^k$ then $f + g$ and $f.g$ are continuous on X.
   B) A mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ is closed in X for every closed set V in Y.

(OR)

4. A) Let f be a continuous mapping of a compact metric space (X, d_1) into a metric space (Y, d_2). Then f is uniformly continuous on X.
   B) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then $f(E)$ is connected.

UNIT-III

5. A) State and Prove Chain Rule for Differentiation.
   B) If $C_0 + \frac{C_1}{2} + \cdots\cdots + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0$, where $C_0, \cdots\cdots\cdots\cdots, C_n$ are real constants, prove that the equation $C_0 + C_1 x + \cdots\cdots\cdot + C_{n-1}x^{n-1} + C_n x^n = 0$ has at least one real root between 0 and 1.

(OR)

6. A) Suppose f is real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then there is a point $x \in (a, b)$ such that $f''(x) - \lambda$.
   B) State and Prove Taylor's Theorem.

UNIT-IV

7. A) Let $f: [a, b] \to \mathbb{R}^k$ and let f be differentiable at x_0 $(a < x_0 < b)$.
   If $a < \alpha_n < x_0 < \beta_n < b$ for $n = 1, 2, 3, \cdots\cdots\cdots\cdots\cdots$ and $\alpha_n \to x_0$, $\beta_n \to x_0$ as $n \to \infty$

then

$$\lim_{n\to\infty} \frac{f(\beta_n) - f(\alpha_n)}{\beta_n - \alpha_n} = f'(x_0).$$

B) Suppose f' is continuous on $[a, b]$ and $\epsilon > 0$. Prove that there exists $\delta > 0$ such that $\left|\frac{f(t)-f(x)}{t-x} - f'(x)\right| < \epsilon$, whenever $0 < |t - x| < \delta, a \le x \le b, a \le t \le b$.

(OR)

8. A) If f is continuous on $[a, b]$ then $f \subset R(\alpha)$on $[a, b]$.
   B) If $f(x) = 0$ for all irrational x and $f(x) = 1$ for all rational x prove that $f \notin R(\alpha)$ on $|a, b|$ for any $a < b$.

UNIT-V

9. A) If $f_1, f_2 \in R(\alpha)$ on $[a, b]$ and $f_1(x) \le f_2(x)$ on $[a, b]$ then $\int_a^b f_1\, d\alpha \le \int_a^b f_2\, d\alpha$.
   B) If $f \in R(\alpha)$ on $[a, b]$ and if $|f(x)| \le M$ on $[a, b]$ then

$$\left|\int_a^b f\, d\alpha\right| \le M[\alpha(b) - \alpha(a)].$$

(OR)

10. A) If γ' is continuous on $[a, b]$ then γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

    B) Let $\gamma_1, \gamma_2, \gamma_3$ be curves in the complex plane defined on $[0, 2\pi]$ by $\gamma_1(t) = e^{it}$, $\gamma_2(t) = e^{2it}$, $\gamma_3(t) = e^{2\pi it.sin\,(1/t)}$. Show that these curves have the same range, that γ_1 and γ_2 are rectifiable, that the length of γ_1 is 2π, that the length of γ_2 is 4π, and that γ_3 is not rectifiable.

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(103MA24)

M.Sc DEGREE EXAMINATIONS, Model QP
Mathematics - First Semester
DIFFERENTIAL EQUATIONS

Time : Three hours **Maximum : 70 marks**

Answer **ONE** question from Each Unit. (5 x 14 = 70)

UNIT-I

1. A) Solve the equation $y' - 2xy = xy^2$.

   B) Consider the homogeneous equation $y' + a(x)y = 0$, where $a(x)$ is a continuous function on $-\infty < x < \infty$, which is periodic function with period $\eta > 0$.

   a) Let ϕ be a non-homogeneous trivial solution and let $\psi(x) = \phi(x + \eta)$. Show that ψ is a solution.

   b) Show that there is a constant c such that $\phi(x + \eta) = c\phi(x)$, for all $x \in X$. Show that $c = \exp\left(-\int_0^n a(t)dt\right)$.

(OR)

2. A) State and Prove Existence Theorem.

   B) Find the solutions of initial value problems

   i) $y'' - 2y' - 3y = 0,\ y(0) = 0,\ y' = 0$

   ii) $y'' + 10y = 0,\ y(0) = \pi, y' = \pi^2$.

UNIT-II

3. Let $b(x)$ be continuous function on an interval I. Every solution ψ of $L(y) = y'' + a_1y' + a_2y = b(x)$ on I can be written as $\psi = \psi_p + c_1\phi_1 + c_2\phi_2$, where ψ_p is a particular solution. ϕ_1, ϕ_2 are two linearly independent solutions of $L(y) - y'' + a_1y' + a_2y - b(x)$ and c_1, c_2 are constants. A particular solution ψ_p is given by

$$\psi_p(x) = \int_{x_0}^{x} \frac{[\phi_1(t)\phi_2(x) - \phi_1(x)\phi_2(t)]b(t)}{w(\phi_1\phi_2)(t)} dt.$$

   Conversely every such ψ is a solution of $L(y) = y'' + a_1y' + a_2y = b(x)$.

(OR)

4. A) If $\phi_1, \ldots\ldots, \phi_n$ are n solutions $L(y) = 0$ on an interval I, they are linearly independent there if and only if $W(\phi_1, \ldots\ldots, \phi_n)(x) \neq 0$ for all x in I.

   B) Find the solution of a non-homogeneous differential equation $y''' + y'' + y' + y - 1$ satisfying $\phi(0) - 0, \phi'(0) - 1, \phi''(0) - 0$.

UNIT-III

5. A) State and Prove Uniqueness Theorem.

   B) The equation $y_1 + a(x)y = 0$ has a solution $\phi(x) = \exp\left(-\int_{x_0}^{x} a(t)\, dt\right)$. This suggest that trying to find a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ of the form $\left(-\int_{x_0}^{x} a(t)\, dt\right)$ where $p(x)$ is a function to be determined. Show that ϕ is a solution of $L(y) = 0$ if and only if p satisfying the first order non-linear equation

$y' - -a_1(x)y - a_2(x).$

(OR)

6. A) Solve the following problems:
   i) $y'' + \frac{1}{4x^2}y = 0, \phi_1(x) = \sqrt{x}$ for $x > 0$.
   ii) $x^2y'' - xy' + y = 0, \phi_1(x) = x$.

   B) One solution of $x^2y'' - xy' + y = 0$ for $x > 0$ is $\phi_1(x) = x$ then find the solution ϕ of $x^2y'' - xy' + y = x^2$.

UNIT-IV

7. A) Find all the solutions of the following equations for $|x| > 0$
   i) $x^2y'' + xy' + 4y = 1$
   ii) $x^2y'' - 3xy' + 5y - 0$
   iii) $x^2y'' + xy' - 4\pi y = x$

   B) Compute the indicial polynomial and their roots for the following equation: $x^2y'' + \sin x y' + \cos x y = 0$.

(OR)

8. Solve the Bessel's Differential Equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$.

UNIT-V

9. A) Let M and N be two real valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \le a,\ |y - y_0| \le b$. Then, the equation $M(x, y)dx + N(x, y)dy - 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

   B) **i)** Show that the function f given by $f(x, y) = y^{\frac{1}{2}}$ does not satisfy Lipchitz condition on $R: |x| \le 1, 0 \le 1$.

   ii) Show that the function f defined in (i) satisfies Lipchitz condition on any rectangle of the form $R: |x| \le a,\ b \le y \le c\ \ (a, b, c > 0)$

(OR)

10. A) Consider the initial value problem $y' = 1 + y^2,\ y(0) = 0$
    i) Using separation of variables, find the solution ϕ of this problem. On what interval does ϕ exist?
    ii) Show that the successive approximations $\phi_0, \phi_1, \phi_2, \ldots$ exist for all real x.
    iii) Show that $\phi_p(x) \to \phi(x)$ for each x satisfying $|x| \le \frac{1}{2}$.

    B) Let $f(x, y) = \frac{\cos y}{1 - x^2}\ (|x| < 1)$
    i) Show that f satisfies Lipchitz condition on every strip $S_a: |x| \le a\ (0 < a < 1), |y| < \infty$.
    ii) Show that every initial value problem $y' - f(x, y), y(0) - y_0\ (|y| < \infty)$ has a solution which exists for $|x| < 1$.

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(104MA24)

M.Sc DEGREE EXAMINATIONS, Model QP
Mathematics - First Semester
TOPOLOGY

Time : Three hours **Maximum : 70 marks**

Answer ONE question from each unit (5x14=70)

UNIT-I

1. (a) Let X be an arbitrary non-empty set, show that the function d defined by

$$d(x,y) = \begin{cases} 0 \text{ } if \text{ } x = y, \\ 1 \text{ } if \text{ } x \neq y. \end{cases}$$

For all $x, y \in R$, is a metric on X.

(b) Show that a subset of a metric space is bounded iff it is non empty and is contained in some closed sphere.

(or)

2. (a) Let (X, d) be a metric space. A subset G of x is open if, and only if, it is a union of open spheres.

(b) State and prove Cantor's intersection theorem.

UNIT-II

3. (a) Let X be a non-empty set and let T(X) be the class of all topologies on X. Let $\leq$ on T(X) be

defined by $T_1 \leq T_2$ iff $T_1 \subset T_2$ for $T_1, T_2 \in T(X)$. Then $(T(X), \leq)$ in a complete lattice.

(b) Let (X, τ) be topological space and $A \subseteq X$. Then $\bar{A} = \{x/x \in X$ and every neighbored of

x intersects $A\}$

(or)

4.(a) Prove that every separable metric space is second countable.

(b) Let X be a non-empty set. Then the collection of all topologies on X forms a complete lattice under the relation "is weaker than".

UNIT-III

5. (a) Prove that any closed subspace of a compact space is compact.

(b) Prove that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

(or)

6. (a) State and prove Lebesgue covering theorem.

(b) Let (X, d) be a compact metric space and $F \subseteq C(X, \mathbb{C})$. If F is compact then F is equicontinuous.

UNIT-IV

7. (a) Every compact Hausdorff space is normal.

(b) State and prove Urysohn's Lemma.

(or)

8.State and prove Tietze extension Theorem.

UNIT-V

9. (a) The product of any non-empty class of connected spaces isconnected.

(b) Prove that the spaces $\mathbf{R}^n$ and C^n are connected.

(or)

10. State and prove Urysohn's theorem.

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(105MA24)

M.Sc DEGREE EXAMINATIONS, Model QP
Mathematics - First Semester
ADVANCED DISCRETE MATHEMATICS

Time : Three hours **Maximum : 70 marks**

Answer ONE question from each UNIT.
All questions carry equal marks.

UNIT-1 (14 Marks (7 + 7))

1(a). What do You mean by Implication and Biconditional. Explain by giving one example to each. Also write truth tables for both Implication and Biconditional.

1(b). What do you mean by 'well formed formula'. Give five examples for well formed formulas.

(OR)

2 (a). Show that (not through the truth tables) the two statements

$[(p \Rightarrow q) \wedge (p \Rightarrow r)]$ and $[p \Rightarrow (q \wedge r)]$ are equivalent.

2(b). Find the principal conjunctive normal form (PCNF) of the formula

$\left[p \rightarrow (q \wedge r)\right] \wedge \left[\overline{p} \rightarrow (\overline{q} \wedge \overline{r})\right]$ **is π (1, 2, 3, 4, 5, 6) (Here we use standard notation π for product).**

UNIT-2 (14 Marks (7 + 7))

3(a). "If there was a party, then catching the train was difficult. If they arrived on time then catching the train was not difficult. They arrived on time. Therefore there was no party." Show that the statement constitutes a valid argument.

3(b). Find the negation of the given expression: "$(x)\,(E(x) \rightarrow S(x))$".

(OR)

4 (a). Prove the following statement (transitivity) by using the rules of Inference:

$$(x)\,(P(x) \rightarrow Q(x)) \wedge (x)\,(Q(x) \rightarrow R(x)) \Rightarrow (x)\,(P(x) \rightarrow R(x))$$

4(b). Prove that "$\exists x\,(M(x))$" follows logically from the premises.

$$(x)\,(A(x) \rightarrow M(x)) \text{ and } \exists x\, A(x)$$

UNIT-3 (14 Marks (7 + 7))

5(a). Define State Machine.

Draw the state diagram for the machine given by the table.

	δ		θ	
	0	**1**	**0**	**1**
s_1	s_1	s_2	x	y
s_2	s_2	s_4	z	x
s_3	s_3	s_3	x	y
s_4	s_1	s_2	y	z

5(b). If f is a state homomorphism from $M_1 = (\zeta_1, \mathcal{J}, \delta_1)$ into $M_2 = (\zeta_2, \mathcal{J}, \delta_2)$, then prove that $f(\delta_1^*(s, w)) = \delta_2^*(f(s), w)$ for all $w \in \mathcal{J}^*$.

(OR)

6(a). Let $M = (\zeta, \mathcal{J}, O, \delta, \theta)$ be an *i*/o-machine. Then prove that there exists a state output machine $M_1 = (\zeta_1, \mathcal{J}, O, \delta_1, \rho)$ and a one-one function f from ζ into ζ_1 such that $\beta_s = \beta_{f(s)}$ for all $s \in \zeta$.

6(b). Find ζ_R and hence the reduced machine M_R for the machine given by the table.

State s	δ		θ	
	0	**1**	**0**	**1**
s_1	s_2	s_5	1	0
s_2	s_5	s_5	1	1
s_3	s_1	s_8	1	1
s_4	s_8	s_2	1	0
s_5	s_6	s_5	1	1
s_6	s_1	s_5	1	1
s_7	s_2	s_3	1	0
s_8	s_3	s_5	1	1

UNIT-4 **(14 Marks (7 + 7))**

7 (a). Prove that an Algebraic Lattice can be turned into a Lattice ordered set.

7(b). Define Lattice Homomorphism.

Verify that the homomorphic image $f(L)$ is a sublattice of M where $f : L \to M$ is a lattice homomorphism.

(OR)

8(a). Draw the diagrams for Diamond Lattice, and Pentagon Lattice. Show that these two lattices are not distributive lattices.

8(b). Find the c.n.f for $p = x_1^1 x_2 + x_1 x_2^1$.

UNIT- 5 **(14 Marks (7 + 7))**

9(a). Define Boolean Homomorphism.

If $f : B_1 \to B_2$ is a Boolean homomorphism, then prove the following (i) to (iii):

(i) $f(0) = 0$, $f(1) = 1$;

(ii) for all $x, y \in B_1$, $x \le y \Rightarrow f(x) \le f(y)$; and

(iii) $f(B_1)$ is a Boolean subalgebra of B_2.

9(b). Let M be an ideal of a Boolean algebra B, then prove that the following two conditions are equivalent:

(i) M is a maximal ideal.

(ii) $b \in B \Rightarrow b \in M$ or $b^1 \in M$, but not both.

(OR)

10(a). Prove that a polynomial $p \in P_n$ is equivalent to the sum of all its prime implicants.

10(b). (i). Draw switching circuit which represent the Boolean expression: $(x_1 \wedge x_2) \vee (x_1 \wedge x_3)$.

(ii). Draw the gating network for the Boolean expression given here.

$$p = x_1x_2x_3 + x_1x_2^1x_3^1 + x_1^1x_2x_3^1 + x_1^1x_2^1x_3.$$