(DM01)

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M.Sc. (Previous) DEGREE EXAMINATION, DEC. – 2016

First Year

MATHEMATICS

Algebra

Time : 3 Hours **Maximum Marks : 70**

Answer any Five questions
All questions carry equal marks.

Q1) a) If G is an abelian group of order o(G) and p is a prime number such that $p^{\alpha} / \mathrm{o(G)},\ p^{\alpha+1} / \mathrm{o(G)}$ then prove that G has a subgroup of order p^{α}.

b) State and prove the Cayley's theorem.

Q2) a) Define an automorphism of a group. If G is a group, then prove that A(G), the set of automorphism of G, is also a group.

b) State and prove the Cauchy's theorem for abelian groups.

Q3) a) Define the Kernel of a group homomorphism. If ϕ is a homomorphism of G onto $\overline{\mathrm{G}}$ with Kernel K, then show that K is a normal subgroup of G.

b) Prove that every permutation can be uniquely expressed as a product of disjoint cycles.

Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ as the product of disjoint cycles.

Q4) a) State and prove the unique factorization theorem.

b) Show that every integral domain is a field.

Q5) a) State and prove the Einestein's criterion.

b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

Q6) a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

b) Prove that the number e is transcendental.

Q7) a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non trivial common factor.

b) If L is a finite extension of K and K is a finite extension of F, then prove that L is a finite extension of F. Moreover [L: F] = [L: K] [K: F].

Q8) State and prove the fundamental theorem of Galois theory.

Q9) a) Define a modular lattice. Prove that every distributive lattice is modular but not conversely.

b) Prove that every distributive lattice with more than one element can be represented as a subdirect union of two element chains.

Q10) a) State and prove the Schreier's theorem.

b) Define a Boolean algebra. If B is a Boolean algebra and $a, b \in B$, then show that

i) $(a')' = a$

ii) $(a \wedge b)' = a' \vee b'$

iii) $a = b \Leftrightarrow (a \wedge b') \vee (a' \wedge b) = b.$

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(DM02)

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M.Sc. (Previous) DEGREE EXAMINATION, DEC. – 2016

First Year

MATHEMATICS

Analysis

Time : 3 Hours **Maximum Marks : 70**

Answer any Five questions
All questions carry equal marks.

Q1) a) Let Y be a subset of the metric space X. Prove that a subset E of Y is open relative to Y, if and only if $E = Y \cap G$ for some open set G of X.

b) Define a compact set. Prove that compact subsets of metric spaces are closed.

Q2) a) If P is a non empty perfect set in R^K, then prove that P is uncountable.

b) Prove that every K-cell is compact.

Q3) a) Prove that (i) Every convergent sequence in any metric space is a Cauchy's sequence and (ii) Every Cauchy sequence in R^K converges.

b) Show that the Cauchy product of two absolutely convergent series converges absolutely.

Q4) a) State and prove a necessary and sufficient condition for a mapping f of a metric space X into a metric Y to be continuous on X.

b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then show that f(E) is connected.

Q5) a) If f is continuous on $[a, b]$, then prove that $f \in R(\alpha)$ on $[a, b]$.

b) State and prove the fundamental theorem of integral calculus.

Q6) a) Let α increase monotonically and $\alpha' \in R$ on $[a, b]$. If f be a bounded real function on $[a, b]$ then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$.

b) Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Then prove that $h \in R(\alpha)$ on $[a, b]$.

Q7) a) State and prove the Cauchy's criterion for uniform convergence of a sequence of functions defined on a set E.

b) Suppose K is compact and (i) $\{f_n\}$ is a sequence of continuous functions on K (ii) $\{f_n\}$ converges pointwise to a continuous function f on K and (iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in \mathrm{K}, n = 1, 2, 3, ...$ Then show that $f_n \to f$ uniformly on K.

Q8) a) Show that there exists a real continuous function on the real line which is nowhere differentiable.

b) If K is a compact metric space, if $f_n \in \mathscr{C}(\mathrm{K})$ for n = 1, 2, 3, ... and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

Q9) a) State and prove the Lebesgue's monotone convergence theorem.

b) Let f and g be measurable real-valued functions defined on X. For a real valued continuous function F on R^2, put $h(x) = \mathrm{F}(f(x), g(x)), x \in \mathrm{X}$. Then show that h is measurable and in particular $f + g, fg$ are measurable.

Q10) a) State and prove Lebesgue's dominated convergence theorem.

b) State and prove the Riesz-Fischer theorem.

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M.Sc. (Previous) DEGREE EXAMINATION, DEC. – 2016

First Year

MATHEMATICS

Complex Analysis & Special Functions & Partial Differential Equations

Time : 3 Hours **Maximum Marks : 70**

Answer any Five questions.
Choosing at least Two from each section.
All questions carry equal marks.

SECTION – A

Q1) a) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$.

b) Prove that $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n P_n(x)$

Deduce the first three Legendre polynomials.

Q2) a) With the usual notation, prove that

i) $J_{-n}(x) = (-1)^n J_n(x)$

ii) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$

b) Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$, n being on integer.

Q3) a) State and prove the necessary and sufficient condition for the integrability of the equation $P\,dx + Q\,dy + R\,dz = 0$

b) Solve the equation

$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$

Q4) a) Solve $(D^2 - D^1)z = 2y - x^2$

b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx.\cos ny$

Q5) a) Solve $r - t\cos^2 x + p\tan x = 0$ by Monge's method.

b) Find a surface passing through two lines $x = z = 0, z - 1 = x - y = 0$ satisfying $r + 4s + 4t = 0$.

SECTION - B

Q6) a) For a given power series $\sum_{n=0}^{\infty} a_n (z-a)^n$ define $0 \le \mathrm{R} < \infty$ by $\frac{1}{\mathrm{R}} = \mathrm{Lim\,sup} |a_n|^{\frac{1}{n}}$. Then prove that (i) if $|z - a| < \mathrm{R}$, then the series converges absolutely (ii) if $|z - a| > \mathrm{R}$, the series diverges (iii) if $0 < r < \mathrm{R}$, then the series converges uniformly on $\{z \| z | \le r\}$.

b) Distinguish between differentiability and analycity of a function. Show that $f(z) = \overline{z}$ is not analytic.

Q7) a) State and prove the Cauchy's theorem.

b) Discuss the mapping properties of cos*z* and sin*z*.

Q8) a) Find an analytic function $f : \mathrm{G} \to c$ where $\mathrm{G} = \{z \mid \mathrm{Re}\, z > 0\}$ such that $f(\mathrm{G}) = \mathrm{D} = \{z : |z| < 1\}$.

b) State and prove the open mapping theorem.

Q9) a) State and prove the Morera's theorem.

b) Evaluate $\int_r \frac{2z+1}{z^2+z+1} dz$ where r is the circle $|z| = 2$

Q10) a) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx$ using the theory of residues.

b) State the residue theorem. Using this theorem evaluate $\int_0^{\pi} \frac{d\theta}{3+2\cos\theta}$

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M.Sc. (Previous) DEGREE EXAMINATION, DEC. – 2016

First Year

MATHEMATICS

Theory of Ordinary Differential Equations

Time : 3 Hours **Maximum Marks : 70**

Answer any Five questions
All questions carry equal marks

Q1) a) If $\phi_1, \phi_2, \ldots, \phi_n$ are n solutions of $L(y) = 0$ on an internal I, prove that they are linearly independent there if and only if $W(\phi_1, \phi_2, \ldots, \phi_n)(x) \neq 0 \forall x$ in I.

b) Find the two solutions ϕ_1, ϕ_2 of the equation $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0,\ x > 0$ satisfying $\phi_1(1) = 1, \phi_2(1) = 0, \phi'_1(1) = 0, \phi'_2(1) = 1$.

Q2) a) Find two linearly independent power series solutions in powers of x for the equation $y'' + 3x^2y' - xy = 0$.

b) Obtain the Rodrigue's formula for the Legendre's differential equation.

Q3) a) Let M, N be two real valued functions having continuous first partial derivatives on some rectangle $R : |x - x_0| \leq a, |y - y_0| = b$. Then show that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

b) Find an integrating factor of the equation $(\cos x)\cos y\, dx - 2(\sin x)\sin y\ dy = 0$ and hence solve it.

Q4) a) Consider $y' = 3y + 1, y(0) = 2$. Show that the successive approximations $\phi_0, \phi_1, \ldots$ exist for all x. Compute the first four approximations $\phi_0, \ldots, \phi_3$ to the solution.

b) Show that the function f given by $f(x, y) = x^2 |y|$ satisfy a Lipschitz condition on $R : |x| \leq 1, |y| \leq 1$. Show that $\frac{\partial y}{\partial x}$ does not exist at $(x, 0)$ if $x \neq 0$.

Q5) a) Compute a solution for the system $y_1' = 3y_1 + 4y_2, y_2' = 5y_1 + 6y_z$.

b) Show that all real valued solutions of the equation $y'' + \sin y = b(x)$ exist for all real x, where $b(x)$ is continuous for $-\infty < x < \infty$.

Q6) a) State and prove the existence theorem for linear systems and establish the uniqueness.

b) Suppose $y = (8+i, 3i, -2), z = (i, -i, 2)$ and $w = (2+i, 0, 1)$ be vectors in C_3. Compute y–z. Verify whether these vectors are associative and commutative.

Q7) a) Find the solution of the Riccati equation $W^1 - W^2 - 1 = 0$.

b) Find a function Z(x) such that $Z(x)[y'' + 3y' + 2y] = \frac{d}{dx}[K(x)y' + m(x)y]$

Q8) a) Compute the Green's function for the equation $y'' - 4y' + 3y = x, (-\infty < x < \infty)$. Hence find the general solution.

b) Derive an adjoint equation for $L(y) = y' - Ay = 0$ where A is a $n \times n$ matrix. Obtain a condition for the operator L to be self adjoint.

Q9) a) State and prove the Sturm comparison theorem.

b) Put the differential equation $y'' + f(t)y' + g(t)y = 0$ into self-adjoint form.

Q10) a) State and prove the Liapunov's inequality.

b) Derive a condition for the equation $P_0 u'' + P_1 u' + P_2 u = 0$ to be self-adjoint.

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