(DMSTT01)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, DEC. – 2016

First Year

STATISTICS

Probability and Distribution Theory

Time : 3 Hours **Maximum Marks : 70**

Answer any five questions.
All questions carry equal marks.

Q1) a) Explain (i) statistical independence and (ii) convolution of random variables. Give examples.

b) Define distribution. State and prove its properties.

Q2) a) Give the axiomatic definition of probability. State its properties. State and prove Kolmogorov in equality.

b) Define characteristic function. State its properties. State and prove inversion theorem.

Q3) a) State and prove Chebyshev's WLLN and Khintchine's WLLN.

b) Examine whether Kolmogorov's SLLN holds for the sequence of *i.i.d* random variables with $p.d.f : f(x) = \frac{e}{x^2 \log x}, x > e.$

Q4) a) Explain the types of convergence. Show that convergence in distribution is weaker than convergence in probability and hence convergence *a.s*.

b) State and prove Lindberg-Levy form of central limit theorem.

Q5) a) Define Hypergeometric distribution. Obtain its *m.g.f* and hence its mean and variance.

b) Derive compound binomial distribution.

Q6) a) State and prove the reproductive property of Poisson random variables.

b) Derive compound Poisson distribution.

Q7) a) Define log-normal distribution. State and prove its reproductive property.

b) Define logistic distribution. Obtain its *m.g.f*.

Q8) a) Define Weibull distribution. Obtain its characteristic function. Hence obtain its mean and variance.

b) Define Laplace distribution. Obtain its characteristic function and hence its mean and variance.

Q9) a) Derive the *p.d.f* of t-statistic.

b) Obtain the joint *p.d.f* of j^{th} and k^{th} order statistics. Find the marginal *p.d.f* of r^{th} order statistics.

Q10) a) Derive the *p.d.f* of non-central Chi-square.

b) Derive the *p.d.f* of F-statistics with (m,n) degrees of freedom.

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(DMSTT02)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, DEC. – 2016

First Year

STATISTICS

Statistical Inference

Time : 3 Hours **Maximum Marks : 70**

Answer any five questions.
All questions carry equal marks.

Q1) a) State and prove Rao-Blackwell theorem.

b) State and prove the uniqueness property of the MVU estimator.

Q2) a) Derive the Cramer-Rao lower bound for the parameter θ in $f(x:\theta)=\frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right), x \geq 0, \theta > 0.$

b) State and prove Lehmann-Scheffe theorem.

Q3) a) Explain *m.l* method of estimation. State the properties of *m.l.* estimators. Find *m.l.* estimators of θ_1 and θ_2 in $f(x:\theta_1,\theta_2)=\frac{1}{\theta_1}\exp\left(-(x-\theta_2)/\theta_1\right), \theta_2 \leq x < \infty, \theta_1 > 0$ and $-\infty < \theta_2 < \infty$.

b) Define CAN and CAUN estimators. Describe the construction of CAN estimators based on percentiles.

Q4) a) Obtain confidence limits for the parameter μ in N(μ, 1) with confidence coefficient $(1-\alpha)$.

b) Describe *m.l.* method of estimation. Obtain *m.l.* estimators of μ and σ^2 based on a random sample of size *n* from N(μ, σ^2).

Q5) a) State and prove Neyman-Pearson Lemma.

b) Consider *n* Bernoullian trials with probability of success *p* for each trial. Derive the likelihood ratio test for $H_0 : p = p_0$ against $H_1: p > p_0$ and $H_1: p < p_0$. Show that they are identical with the corresponding UMP tests.

Q6) a) Explain likelihood ratio test procedure. Obtain the asymptotic distribution likelihood ratio test.

b) Obtain the likelihood ratio test for testing H_0: $\theta = \theta_0$ against the alternative H_1: $\theta > \theta_0$ in the case of $N(\theta, \sigma^2)$, σ^2 is known. Show that it is identical with the corresponding UMP test.

Q7) a) Explain Wilcoxon signed rank test.

b) Explain in Kolmogorov-Smirnov one sample and two sample tests.

Q8) a) Explain Wilcoxon Mann-Whitney test.

b) Explain sign test.

Q9) a) Describe SPR test and show that it terminates with probability one.

b) Derive SPR test to test the parameter λ of a Poisson distribution. Obtain its OC and ASN functions.

Q10) a) Determine the constants A and B in SPR test.

b) Derive SPR test to test the mean of $N(\mu, \sigma^2)$, σ^2 is known. Obtain OC and ASN functions.

(DMSTT03)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, DEC. – 2016

First Year

STATISTICS

Sampling Theory

Time : 3 Hours **Maximum Marks : 70**

Answer any five questions.
All questions carry equal marks.

Q1) a) What is a sample survey? In what respect is it superior to a census survey? Distinguish between sampling and non-sampling errors.

b) Discuss the main steps involved in a sample survey.

Q2) a) Discuss the basic principles of a sample survey.

b) Explain the organization and functions of NSSO.

Q3) a) Derive the formula for the sample size in the case of continuous data.

b) Explain the allocation problems in stratified random sampling.

Q4) a) Explain the purpose of stratification in sample surveys. Under what conditions is stratified random sampling preferred to simple random sampling and why?

b) Compare simple random sampling and stratified random sampling with proportional and optimum allocations. When does stratification produce large gains in precisions.

Q5) a) What is cluster sampling? Give its applications. Obtain the variance of the estimated mean in cluster sampling with clusters of equal sizes.

b) Explain systematic sampling. What are its advantages and disadvantages? Obtain the relative efficiency of systematic sampling as compared to SRSWOR.

Q6) a) Determine the optimum cluster size so as to minimize the variance for a fixed cost.

b) Explain circular systematic sampling. Derive the variance of the estimated mean in systematic sampling.

Q7) a) Explain Lahiri's method of PPS sampling. Derive the variance of the estimated population total in PPS sampling with replacement.

b) Explain multistage sampling. What are its advantages and disadvantages? Give its applications.

Q8) a) Obtain an unbiased estimate of the population total in PPS sampling with replacement. Derive the variance of the estimated population total.

b) Explain two-stage sampling. What are its applications? Obtain the estimate of variance of the estimated mean in two-stage sampling with equal second stage units.

Q9) a) Derive the conditions under which ratio estimator is BLUE. Compare the ratio estimate with the mean per unit.

b) Obtain the variance of the linear regression estimate with preassigned regression coefficient (b_0). Determine the best choice for b_0.

Q10) a) Define ratio estimates in stratified random sampling and obtain $V(\hat{R}_{rs})$ in the usual notation.

b) Discuss the relative efficiencies of ratio and regression estimates.

(DMSTT04)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, DEC. – 2016

(First Year)

STATISTICS

Design of Experiments

Time : 3 Hours **Maximum Marks : 70**

Answer any five questions.
All questions carry equal marks.

Q1) a) Explain (i) differentiation using matrices (ii) trace of a matrix and (iii) idempotent matrix with examples.

b) Find the characteristic roots and vectors of $A = \begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 2 & 2 & -3 \end{pmatrix}$.

Q2) a) State and prove Cauley-Hamilton theorem.

b) State Cochran's theorem for quadratic forms.

Find the determinant of $A = \begin{pmatrix} 1 & 2 & 5 & 2 \\ -2 & 3 & 0 & -4 \\ 1 & -1 & 0 & 2 \\ 0 & 1 & 4 & 2 \end{pmatrix}$.

Q3) a) Explain the linear model. Define best linear unbiased estimate.

b) State and prove Gauss-Markov theorem.

Q4) a) Explain the generalized linear model. State and prove a necessary and sufficient condition for the estimability of a linear parametric function.

b) State and prove Aitken's theorem.

Q5) a) Explain analysis of variance of one-way classification with equal number of observations.

b) Explain analysis of covariance of two-way classification.

Q6) a) Explain analysis of variance of two-way classification with unequal number of observations.

b) Explain analysis of covariance of one-way classification.

Q7) a) Explain the analysis of RBD.

b) Explain Gracco Latin Square design. Give its analysis.

Q8) a) What are mutually orthogonal LSDs? How do you construct them?

b) Explain the analysis of LSD with a missing row or a missing column.

Q9) a) What are factorial experiments? Explain the analysis of 3^3 factorial experiment.

b) Define BIBD. State and prove its parametric relations.

Q10) a) Explain the analysis of 2^2 factorial experiment.

b) Discuss the intra block analysis of BIBD.