

(DMSTT 01)

M.Sc. DEGREE EXAMINATION,
NOVEMBER 2021.

First Year

Statistics

PROBABILITY AND DISTRIBUTION THEORY

Time : Three hours

Maximum : 70 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) State and prove Borel-Cantelli lemma.
(b) Define continuity axiom probability. Explain.
2. (a) State and prove central limit theorem.
(b) Define mathematical expectation. Explain.
3. (a) Discuss about the Chebyshev and Khintchin laws.
(b) Explain the types of convergence. Prove that almost sure convergence implies convergence in probability.
4. (a) State and prove Kinchine's form of W.L.L.N.
(b) State and prove Kolmogorov's S.L.L.N for i.i.d random variables.
5. (a) Find the m.g.f. of truncated binomial and hence find its mean and variance.
(b) Derive the compound binomial distribution.
6. (a) Write the properties of interrelations of multinomial.
(b) Discuss about the m.g.f. and probability generating functions.
7. (a) Define log-normal distribution. Obtain r^{th} row moment. Hence obtain its variance.
(b) Explain about the logistic of the distribution.

8. (a) Derive m.g.f. of Laplace distribution.
- (b) The joint p.d.f. of two dimensional random variable (x,y) is
- $$f(x,y) = \begin{cases} 2; & 0 < x < 1, 0 < y < x \\ 0; & elsewhere \end{cases}$$
- (i) Find the marginal density function of x and y .
- (ii) Find the conditional density function of y given $x = x$.
9. (a) Explain the central chi-square distribution and find its mean and variance and explain their populations.
- (b) Let $f(x,y)$ be jointly distributed with the density
- $$f(x,y) = \frac{y}{(1+x)} (xy)^{\left[\frac{-y}{1+x} \right]}; x, y > 0 \text{ find } E(Y/X = x).$$
10. (a) Let y_1, y_2, \dots, y_n be the order statistics form, a random sample of size n from an exponential distribution. Obtain the distributed of y_1 and y_n .
- (b) Explain non central F-distribution find its mean variance. Explain their properties.
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STATISTICAL INTERFACE

Time : Three hours

Maximum : 70 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) State and prove Cramer-Rao inequality
(b) Define a minimum variance unbiased estimator. Show that sample variance is an unbiased estimator of the population variance.
2. (a) State and prove scheff theorem.
(b) Let x_1, x_2, \dots, x_n be a random sample from the distribution with p.d.f $f_\theta(x) = \frac{1}{\beta - \alpha}$, if $\alpha < x < \beta$ where $\theta = (\alpha, \beta)$ and $0 < \alpha < \beta$. Obtain MVU estimator of $\frac{\alpha + \beta}{2}$ and $\beta - \alpha$.
3. (a) Explain ML method of estimation. Find the ML estimator of θ for random samples from the distribution $f_\theta(x) = \frac{1}{\theta} \exp\left(\frac{-x}{\theta}\right)$, $0 \leq x \leq \infty$.
(b) What is interval estimator? Explain with an example.
4. (a) Explain (i) consistency and (ii) CAUN estimator
(b) Obtain confidence limits for the parameter μ in $N(\mu, 1)$ with confidence coefficient $(1 - \alpha)$.
5. (a) Discuss about the concept of monotone likelihood ratio.
(b) State and prove Neyman-Pearson lemma.

6. (a) Explain about the non-randomised and randomised test.
(b) Consider n Bernoulli trials with probability of success P for each trial. Derive the likelihood ratio test for $H_0: P=P_0$ against $H_1: P>P_0$ and $H_1: P<P_0$. Then show that they are identical with UMP tests.
 7. (a) Explain Wilcoxon-Mann - Whitney U-test.
(b) What are the applications of wilcoxon test? Explain its procedure.
 8. (a) What is median test? Explain.
(b) Explain about clearly Wilcoxon signed rank.
 9. (a) What is SPRT? Explain the procedure of SPRT.
(b) Explain OC and ASN functions of the SPRT.
 10. (a) Derive SPRT to test the parameter λ of a Poisson distribution about its OC and ASN functions.
(b) Explain SPRT in the case of binomial distribution.
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(DMSTT 03)

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First Year

Statistics

SAMPLING THEORY

Time : Three hours

Maximum : 70 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) What is the differences between enumeration survey and sample survey? Explain the features of sample survey.
(b) Write about sampling errors and give its sources.
2. (a) What are sampling and non-sampling errors faced by the researcher? Explain.
(b) What are the responsibilities of central statistical organisation?
3. (a) Compare the variances of systematic stratified and simple random sampling schemes in population with linear trend.
(b) Define simple random process. Explain its merits and demerits.
4. (a) Why stratification is important is sample determination? Explain.
(b) Distinguish between simple random sampling and systematic random sampling.
5. (a) Define cluster sampling. Deduce mean and variance.
(b) How do you estimate mean and variance with systematic sampling?
6. (a) Explain the concept of circular systematic sampling with an example.
(b) What are the features of cluster sampling? Explain.

7. (a) Explain PPS sampling with and without replacement.
(b) Write about two stage sampling. Deduce mean and variance of two-stage sampling.
 8. (a) Explain two examples where we can use multi-stage sampling.
(b) Explain double sampling. Obtain an estimate of variance of an unbiased estimate of population mean in double sampling for stratification.
 9. (a) Explain ratio method of estimation. Obtain an exact expression for bias and upper bound for bias of the ratio estimator.
(b) Explain ratio estimate is stratified sampling.
 10. (a) Compare ratio estimate with mean per unit. State and prove the conditions under which ratio estimator is a BLUE.
(b) Explain double sampling for regression estimation and its uses.
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Statistics

DESIGN OF EXPERIMENTS

Time : Three hours

Maximum : 70 marks

Answer any FIVE questions.

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1. (a) Find the characteristic roots and vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
- (b) Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are =1 each. Find the eigen values of A^{-1}
2. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.
- (b) Verify that the following matrix is orthogonal if $A = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$.
3. (a) State and prove Gauss – Markov theorem.
(b) Discuss about the linear models with examples.
4. (a) What is best linear unbiased estimate? Explain.
(b) Explain generalised linear models.
5. (a) Explain two-way ANOVA with an examples.
(b) What are random effect models? Explain analysis of three way classification.

6. (a) Define two-way, three-way classification.
(b) Explain analysis of co-variance of two-way classification.
 7. (a) What is an experiment? Explain the formulation of hypothesis in experiment.
(b) What is randomized block design? Explain.
 8. (a) Describe missing plot technique? Explain its application.
(b) What is Latin squares? Explain in detail.
 9. (a) What is the importance of factorial experiments? Explain.
(b) Discuss in brief about intra block analysis of BIBD.
 10. (a) Define BIBD. Derive its parametric relations and point out different types of BIBD.
(b) Explain the intra block analysis of PBIBD.
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