(DM01)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2017

First Year

MATHEMATICS

Algebra

Time : 3 Hours **Maximum Marks: 70**

Answer any five of the following

All questions carry equal marks.

Q1) a) State and prove the Cayley's theorem.

b) If ϕ is a homomorphism of a group G into $\bar{G}$, then show that

i) $\phi(e) = \bar{e}$, the unit element of $\bar{G}$.

ii) $\phi(x^{-1}) = [\phi(x)]^{-1}$ for all $x \in G$.

Q2) a) Suppose G is a group and that G is the internal direct product of $N_1, N_2, \ldots, N_n$. If $T = N_1 \times N_2 \times \ldots \times N_n$ then prove that G and T are isomorphic.

b) If p is a prime number and $p^{\alpha} \mid O(G)$, then prove that G has a subgroup of order p^{α}.

Q3) a) State and prove the Cauchy's theorem for abelian groups.

b) Define a permutation. Resolve the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 2 & 5 & 6 & 4 & 8 & 7 \end{pmatrix}$ into disjoint cycles. Given $x = (1, 2)(3, 4)$, $y = (5, 6)(1,3)$, find a permutation a such that $a^{-1} x a = y$.

Q4) a) Prove that every integral domain is a field.

b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

Q5) a) State and prove the Fermat's theorem.

b) State the Eisenstein criterion, prove that the polynomial $1 + x + \ldots + x^{p-1}$, where p is prime number, is irreducible over the field of rational numbers.

Q6) a) If L is a finite extension of K and K is a finite extension of F, then prove that L is a finite extension of F and that [L : F] = [L : K] [K : F].

b) Prove that a polynomial of degree n over a field can have atmost n roots in any extension field.

Q7) a) Prove that it is impossible, by straight edge and compass alone, to trisect 60^{o}.

b) Prove that K is a normal extension of a field F if and only if K is the splitting field of some polynomial over F.

Q8) a) Show that a general polynomial of degree n, $n \geq 5$ is not solvable by radicals.

b) i) Prove that the fixed field of G is a sub field of K.

ii) If K is a finite extension of F, then G(K, F) is a finite group and show that $O(G(K, F)) \leq [K :F]$.

Q9) a) State and prove the Schreier's theorem.

b) Derive the dimensionality equation $d(a \vee b) = d(a) + d(b) - d(a \wedge b)$ for modular lattice.

Q10) a) Prove that every distributive lattice with more than one element can be represented as a sub direct union of two element chains.

b) Define a Boolean algebra and a Boolean ring. Show that a Boolean ring can be converted into a Boolean algebra.

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(DM02)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2017

First Year

MATHEMATICS

Analysis

Time : 3 Hours **Maximum Marks: 70**

Answer any five questions.

All questions carry equal marks.

Q1) a) Let $\{E_n\}$, $n = 1, 2, 3, \ldots$ be a sequence of countable sets and put $S = \bigcup_{n=1}^{\infty} E_n$. Then show that S is countable.

b) Prove that closed subsets of compact sets are compact.

Q2) a) If a set E in R^k has one of the following three properties, then show that it has the other two:

i) E is closed and bounded

ii) E is compact

iii) Every infinite subset of E has a limit point in E.

b) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

Q3) a) Suppose $\{S_n\}$ is monotonic. Then prove that $\{S_n\}$ converges if and only if it is bounded.

b) Show that the product of two convergent series need not converge and may actually leverage.

Q4) a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that f (X) is compact.

b) Define f on R^1 by $f(x) = \begin{cases} 0, & \text{if } x \text{ is irrational} \\ \frac{1}{n}, & \text{if } x \text{ is rational} \end{cases}$

Show that f is continuous at every irrational point and it has a simple discontinuity at rational points.

Q5) a) State and prove a necessary and sufficient condition for a bounded function f to be R – S integrable on $[a, b]$.

b) Suppose that $f \in \square(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi\, f(x)$ on $[a, b]$. Then show that $h \in \square(\alpha)$ on $[a, b]$.

Q6) a) Let f maps $[a, b]$ into R^k and suppose that $f \in \square(\alpha)$ for some monotonically increasing function α on $[a, b]$. Then show that $|f| \in R(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

b) State and prove the fundamental theorem of integral calculus.

Q7) a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

b) If K is compact, $f_n \in \square(K)$ for $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K then prove that

i) f_n is uniformly bounded on K.

ii) $\{f_n\}$ contains a uniformly convergent subsequence.

Q8) State and prove the Weirstrass approximation theorem.

Q9) a) State and prove the Lebesque's monotone convergence theorem.

b) If $\{f_n\}$ is a sequence of measurable functions then prove that the set of points x at which $\{f_n(x)\}$ converges is measurable.

Q10) a) Suppose $f = f_1 + f_2$ where $f_i \in \alpha(u)$ on E$(i = 1, 2)$. Then show that $f \in \alpha(u)$ on E and $\int_E f\, d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu$.

b) If $f \in \mathcal{R}$ on $[a, b]$, then show that $f \in L$ on $[a, b]$ and that

$$\int_a^b f\, dx = R\int_a^b f\, dx$$

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(DM03)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2017

First Year

MATHEMATICS

Complex Analysis & Spe. Functions & Partial Dif. Equ.

Time : 3 Hours **Maximum Marks: 70**

Answer any five questions choosing atleast Two from each section.

All questions carry equal marks.

SECTION- A

Q1) a) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$.

b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

Q2) a) Prove that $\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)z^n P_n$.

b) Find the solution of the Bessel equation of order n and of the first kind, n being a non negative constant.

Q3) a) Prove that the necessary condition for the integrability of the total differential equation $\overline{A} \cdot d\overline{r} = P\,dx + Q\,dy + R\,dz = 0$ is $\overline{A} \cdot$ curl $\overline{A} = 0$.

b) Solve $(yz+2x)dx + (zx-2z)dy + (xy-2y)dz = 0$.

Q4) a) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y-x)$.

b) Solve $y^2 r - 2y\,s + t = p + 6y$ using Monge's method.

Q5) a) Solve the partial differential equation

$(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x+y)$, with the usual notation.

b) Find the general solution of the partial differential equation

$(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^x$.

SECTION- B

Q6) a) Let u and v be real valued functions defined on a region G and suppose that u and v possess continuous partial derivatives. Then prove that f: G $\rightarrow$ C defined by $f(z) = u(z) + i\, v(z)$ is analytic if and only if u and v satisfy the Cauchy – Riemann equations.

b) If $\sum a_n (z-a)^n$ is a given power series with radius of convergence R, then show that $R = \lim \left|\frac{a_n}{a_{n+1}}\right|$, if this limit exist. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$.

Q7) a) Let R be a closed polygon $[1-i, 1+i, -1+i, -1, -i, 1-i]$. Then evaluate $\int_R \frac{1}{z} dz$.

b) State and prove the open mapping theorem.

Q8) a) Define Mobius transformation. Let z_1, z_2, z_3, z_4 be four distinct points in C_∞. Then show that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on a circle.

b) If $r:[a,b] \rightarrow C$ is a piecewise smooth curve then prove that r is of bounded variation and $v(r) = \int_a^b |r'(t)|\, dt$.

Q9) a) State and prove the Laurent series development.

b) Find the Laurent series expansion of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ in the region $2 < |z| < 3$.

Q10) a) Discuss the nature and classification of singularities of a function $f(z)$. Find the nature and location of singularities of

i) $f(z) = e^z / (z-1)^4$

ii) $z^2 - 1 / (z-1)^3$

b) State the residue theorem. Using the residue theory evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx$.

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Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2017

First Year

MATHEMATICS

Theory of Ordinary Differential Equations

Time : 3 Hours **Maximum Marks: 70**

Answer any five questions.

All questions carry equal marks.

Q1) a) If $\phi_1, \phi_2, ..., \phi_n$ are *n* solutions of L(*y*) = 0 on an interval I, prove that they are linearly independent there if and only if $w(\phi_1, \phi_2, ..., \phi_n)(x) \neq 0$ for all x in I, where $L(y) = y^{(n)} + a_1(x)y^{n-1} + ... + a_n(x)y = 0$.

b) If one solution of $y'' - \frac{2}{x^2}y = 0, 0 < x < \infty$ is $\phi_1(x) = x^2$, find the general solution of the equation $y'' - \frac{2}{x^2}y = x$.

Q2) a) Find two linearly independent power series solutions of the equation $y'' - xy = 0$.

b) Compute the solution of $y''' - xy = 0$ which satisfies $\phi(0) = 1$, $\phi'(0) = 0$, $\phi''(0) = 0$.

Q3) a) Let M, N be real valued functions having continuous first partial derivatives on some rectangle R: $|x - x_0| \leq a, |y - y_0| \leq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

b) Find an integrating factor of the equation $\cos x.\cos y \, dx - 2\sin x.\sin y \, dy = 0$ and solve it.

Q4) a) Show that the function ϕ is a solution of the I.V.P. $y' = f(x,y), y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$ on I.

b) Let $f(x,y) = \frac{\cos y}{1-x^2}; (|x|<|)$, show that *f* satisfies a Lipschitz condition on every strip $s_a : |x| \le a$, where 0 < *a* < 1. Show that every initial value problem $y' = f(x,y), y(0) = y_0, (|y_0| < \infty)$ has a solution which exists for | *x* | < |.

Q5) a) Find the solution of $y'' = -\frac{1}{2y^2}$ satisfying $\phi(0) = 1, \phi'(0) = -1$.

b) Find a solution ϕ of the system $y_1^1 = y_1, y_2^1 = y_1 + y_2$ which satisfies ϕ(0)= (1, 2).

Q6) a) State and prove the local existence theorem for the existence of solution to the system: $Y' = f(x,Y), Y(x_0) = Y_0$.

b) Show that all real valued solutions of the equation $y'' + \sin y = b(x)$, where *b* is continuous for $-\infty < x < \infty$, exist for all real *x*.

Q7) a) Find the general solution of the Riccate equation.

b) Find the functions Z(*x*), K(*x*) and *m*(*x*) such that

$$z(x)[x^2 y'' - 2xy' + 2y] = \frac{d}{dx}[k(x)y' + m(x,y)].$$

Q8) a) Show that the Green's function for $L(x) = x'' = 0, x(0) + x(1) = 0$ $x'(0) + x'(1) = 0$ is G(*t*, *s*) = 1− s if $t \le s$ and $G(t,s) = 1-t$ if $t \ge s$.

b) Find the general solution of $y'' - 4y' + 3y = x, (-\infty < x < \infty)$ by computing the particular solution using Green's theorem.

Q9) State and prove the sturm separation theorem.

Q10) a) Express the differential equation $x^2 y'' + x\,y' + (x^2 - n^2)y = 0$ in self-adjoint form.

b) State and prove the Gronwall's inequality.