(DMSTT 01)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, MAY – 2017

First Year

STATISTICS

Probability and Distribution Theory

Time : 3 Hours **Maximum Marks: 70**

Answer any Five questions.

All questions carry equal Marks.

Q1) a) Define distribution function. State and prove its properties.

b) State and prove Kolmogorov's inequality.

Q2) a) State and prove a necessary and sufficient condition for *n* random variables to be independent.

b) State and prove Borel-Cantelli lemma.

Q3) a) Explain modes of convergence. In the usual notation, prove $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{L} X$.

b) State and prove Kolmogorov's strong law of large numbers for independent random variables.

Q4) a) State and prove Levy and Lindberg form of central limit theorem.

b) Determine whether strong law of large numbers holds for the sequence of random variables $P(X_k = \pm 2^k) = 1/2^{2k+1}$, $P(X_k = 0) = 1 - \frac{1}{2^{2k}}$.

Q5) a) Derive compound binomial distribution.

b) Define multinomial distribution. Show that the marginal p.m.f. of each X_i, $i = 1,2, \ldots\ldots k-1$ in a multinomial distribution is binomial.

Q6) a) Derive compound Poisson distribution.

b) Let (X_1, X_2,.... X_{k-1}) have a multinomial distribution with parameters n, p_1, p_2, ... p_{k-1}. Write $Y = \sum_{i=1}^{k} \left(X_i - np_i \right)^2 / np_i$, where $p_k = 1 - p_1 - p_2 - \cdots - p_{k-1}$ and $X_k = n - X_1 - .... - X_{k-1}$. Find E(Y) and V(Y).

Q7) a) Obtain $E(Y^n)$ for a Weibull random variable Y.

b) Define log-normal distribution. Obtain its n^{th} row moment.

Q8) a) Define Laplace distribution. Obtain its m.g.f.

b) Define logistic distribution. Obtain its characteristic function.

Q9) a) Derive the distribution of t.

b) Derive the joint p.d.f. of ($X_{(1)}$, $X_{(2)}$,......,$X_{(n)}$).

Q10) a) Derive the distribution of non-central Chi-square.

b) Obtain the joint p.d.f. of $X_{(j)}$ and $X_{(k)}$, $1 \le j < k \le n$.

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(DMSTT 02)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, MAY – 2017

STATISTICS

Statistical Inference

Time : 3 Hours **Maximum Marks: 70**

Answer any Five questions.

All questions carry equal Marks.

Q1) a) Explain sufficiency. Obtain the general form of the distributions admitting sufficient statistic.

b) State and prove Cramer-Rao inequality.

Q2) a) State and prove Lehmann-Scheffe theorem.

b) Let $X_1, X_2, \ldots .X_n$ be a random sample from the distribution with p.d.f. $f_\theta(x) = \frac{1}{\beta-\alpha}$ if $\alpha < x < \beta$ where $\theta = (\alpha, \beta)$ and $0 < \alpha < \beta < \infty$. Obtain the MVU estimators of $\frac{(\alpha+\beta)}{2}$ and $\beta - \alpha$.

Q3) a) Explain consistency and efficiency. State and prove sufficient conditions for consistency.

b) Find the ML estimator of θ for random sample from $f_\theta(x) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right), 0 \leq x < \infty$.

Q4) a) Explain maximum likelihood method of estimation. State its properties.

b) Explain interval estimation. Let $X_1, X_2, \ldots .X_n$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are both unknown. Obtain the confidence interval for μ.

Q5) a) State and prove Neymann-Pearson lemma.

b) Find UMP tests for testing H_0: $\theta = \theta_0$ against one sided alternatives in $N(\theta, \sigma^2)$ where σ^2 unknown.

Q6) Explain likelihood ratio test. Show that the likelihood ratio test is consistent under the conditions to be specified by you.

Q7) a) Explain :

i) Sign test and

ii) Wilcoxon signed rank test.

b) Explain :

i) Two sample runs and

ii) Median tests.

Q8) a) Explain Wilcoxon – Mann – Whitney U test.

b) Explain Kolmogorov – Smirnor one sample and two sample tests.

Q9) a) Explain Wald's SPRT. Obtain its OC and ASN functions.

b) Determine the SPR test for testing H_0: $\theta = \theta_0$ against H_1: $\theta = \theta_1(\theta_1 > \theta_0)$ where θ is the parameter of a Poisson distribution. Obtain OC and ASN functions of the test.

Q10) a) Show that SPRT terminates with probability one.

b) The random variable X has $N(\mu, \sigma^2)$ where σ^2 known. Develop an SPR test for testing H_0: $\theta = \theta_0$ against H_1: $= \theta_1$. If $\alpha = \beta$ (in the usual notation). Prove that the ASNs under H_0 and H_1 are equal.

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(DMSTT 03)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, MAY – 2017

First Year

STATISTICS

Sampling Theory

Time : 3 Hours **Maximum Marks: 70**

Answer any Five questions

All questions carry equal Marks

Q1) a) Explain the concepts of

i) Sample
ii) Sampling frame and
iii) Complete enumeration survey.

b) Explain the organisation and functions of N.S.S.O.

Q2) a) Distinguish between sampling and non-sampling errors. Describe the sources of non-sampling errors.

b) Explain the organisation and functions of C.S.O.

Q3) a) Explain simple random sampling with and without replacements. In SRSWOR obtain the variance of the sample mean.

b) Explain stratified random sampling. Compare the efficiencies of the Neyman and proportional allocations with that of an unstratified random sample of the same size.

Q4) a) Determine the sample size in sampling from

i) Attribute data and
ii) Variable data.

b) What are the advantages and disadvantages of stratified random sampling? Obtain the variance of sample mean in stratified random sampling.

Q5) a) Explain systematic sampling. What are its merits and demerits? Determine the optimum cluster size for fixed cost.

b) Obtain an unbiased estimator of population mean and its variance in cluster sampling with clusters of equal size.

Q6) a) Explain

i) Systematic sampling and

ii) Circular systematic sampling.

Give their applications two each.

b) Obtain the variance of sample mean in systematic sampling.

Q7) a) Explain the procedures of selecting a p.p.s. sample and their advantages.

b) Obtain the variance of sample mean in two stage sampling with equal number of second stage units.

Q8) a) Obtain the variance of sample total in p.p.s. sampling.

b) Explain two - stage sampling. What are its advantages? Give any two of its applications.

Q9) a) Discuss the relative efficiency of ratio and regression estimates.

b) Obtain the variance of the ratio estimate. Compare it with the estimate based on mean per unit.

Q10) a) Compare the variances of regression estimates in stratified sampling and describe the conditions on the optimum Choices of the regression estimate.

b) Obtain the leading term in the bias of the ratio estimate. Derive the variance of an unbiased ratio estimator of the population total in stratified random sampling.

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(DMSTT 04)

Total No. of Questions : 10] **[Total No. of Pages : 02**

M.Sc. DEGREE EXAMINATION, MAY – 2017

First Year

STATISTICS

Design of Experiments

Time : 3 Hours **Maximum Marks: 70**

Answer any Five questions.

All questions carry equal Marks.

Q1) a) Define:

i) Rank of a matrix.
ii) Inverse of a matrix.
iii) Idempotent matrix and
iv) Trace of a matrix.

Give examples one each.

b) State and prove Cauley-Hamilton theorem.

Q2) a) State Cochran's theorem for quadratic forms. Find the rank of the following matrix: B= $\begin{bmatrix} 5 & 1 & 3 \\ 0 & 0 & 2 \\ 10 & 2 & 4 \end{bmatrix}$

b) Find the characteristic roots and vectors of A = $\begin{bmatrix} 3 & -6 & 6 \\ 2 & -4 & 4 \\ 1 & -2 & 2 \end{bmatrix}$

Q3) a) Explain the

i) Linear model and
ii) Estimable functions.

b) State and prove Gauss-Markov theorem.

Q4) a) Explain the

i) Generalised linear model and
ii) Best linear unbiased estimates.

b) State and prove Aitken's theorem.

Q5) a) Explain the analysis of covariance of two-way classification.

b) Explain the analysis of variance of one-way classification with unequal number of observations.

Q6) a) Explain the analysis of covariance of one-way classification.

b) Explain the analysis of variance of two-way classification with unequal number of observations.

Q7) a) Explain the missing plot technique when some observations are missing.

b) Explain RBD. Obtain the least squares estimates and expectations of means sums of squares.

Q8) a) Explain CRD. Obtain the least squares estimates and expectations of means sums of squares.

b) Explain

i) Graeco – Latin Square Design and

ii) Mutually orthogonal Latin squares design.

Q9) a) Explain the analysis of 2^3 factorial experiment.

b) Explain the interblock analysis of BIBD.

Q10) a) Explain the analysis of 3^2 factorial experiment.

b) Explain the intrablock analysis of BIBD.

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