

Total No. of Questions : 10]

DM01

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2019

(First Year)

MATHEMATICS

Algebra

Time : 3 Hours

Maximum Marks : 70

Answer any Five of the following questions.

All Questions carry equal marks.

- Q1)** a) If ϕ is a homomorphism of G into \bar{G} with kernel K , then prove that K is a normal sub group of G .
- b) If G is abelian of order $O(G)$ and $P^\alpha \mid O(G)$, $P^{\alpha+1} \nmid O(G)$, then prove that there is a unique subgroup of G of order P^α .
- Q2)** a) If G is a group, then show that $A(G)$, the set of automorphisms of G , is also a group.
- b) State and prove Cayley's theorem.
- Q3)** a) If p is prime number and $p^\alpha \mid O(G)$, then prove that G has a subgroup of order p^α .
- b) Let G be a group and suppose that G is the internal direct product of N_1, \dots, N_n . Let $T = N_1 \times \dots \times N_n$. Then prove that G and T are isomorphic.
- Q4)** a) Prove that a finite integral domain is a field.
- b) If ϕ is a homomorphism of R into R' , then prove that (1) $\phi(0) = 0$ (2) $\phi(-a) = -\phi(a)$ for every $a \in R$.
- Q5)** a) If R is unique factorization domain, then show that $R[x]$ is also unique factorization domain.
- b) If R is an integral domain with unit element, prove that any unit in $R[x]$ must already be a unit in R .

- Q6)** a) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F .
- b) If $a \in K$ is algebraic of degree n over F , then prove that $[F(a) : F] = n$.
- Q7)** a) Prove that if α, β are constructible, then so are $\alpha \pm \beta$, $\alpha\beta$ and α/β (when $\beta \neq 0$)
- b) Prove that a circle in F has an equation of the form $x^2 + y^2 + ax + by + c = 0$, with a, b, c in F .
- Q8)** a) Show that the polynomial $p(x) = x^3 - 3x - 3$ over \mathbb{Q} are irreducible and have exactly two non-real roots.
- b) In S_5 , show that $(1\ 2)$ and $(1\ 2\ 3\ 4\ 5)$ generate S_5 .
- Q9)** a) Show that the partially ordered set of subgroups of a cyclic group of prime power is a chain.
- b) Show that any complete Lattice has a zero and an element.
- Q10)** a) Prove that the following two types of abstract systems are equivalent.
- (1) Boolean algebra
 - (2) Boolean ring with identity
- b) Prove that any ring for which there exist a prime p such that $pa = 0$, $a^p = a$ for every a in the ring is commutative.



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DM02

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2019

(First Year)

MATHEMATICS

Analysis

Time : 3 Hours

Maximum Marks : 70

Answer any Five of the following questions.

All Questions carry equal marks.

- Q1)** a) Prove that every infinite subset of a countable set A is countable.
b) Let $\{E_n\}$, $n = 1, 2, 3, \dots$, be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$, then prove that S is countable.
- Q2)** a) Prove that every bounded infinite subset of \mathbb{R}^k has a limit point in \mathbb{R}^k .
b) Prove that every k -cell is compact.
- Q3)** a) The sub sequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X .
b) Suppose $\{s_n\}$ is monotonic. Then prove that $\{s_n\}$ converges if and only if it is bounded.
- Q4)** a) If f is continuous mapping of a compact metric space X into \mathbb{R}^k , then prove that $f(X)$ is closed and bounded.
b) If f is continuous mapping of a compact metric space X into Y . Then f is uniformly continuous on X .
- Q5)** a) If γ' is continuous on $[a, b]$, then prove that γ' is rectifiable, and
$$\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt.$$

b) If $f(x) = 0$ for all irrational x , $f(x) = 1$ for all rational x , prove that $f \notin \mathcal{R}$ on $[a, b]$ for any $a < b$.
- Q6)** a) Prove that $f \in \mathcal{R}$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exist a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.
b) If P^* is a refinement of P , then prove that $L(p, f, \alpha) \leq L(P^*, f, \alpha)$.

Q7)

- a) Prove that the sequence of function $\{f_n\}$ defined on E , converges uniformly on E if and if only for every $\varepsilon > 0$ there exist an integer N such that $m \geq N, n \geq N, x \in E$ implies $|f_n(x) - f_m(x)| \leq \varepsilon$.
- b) Suppose $\{f_n\}$ is a sequence of functions defined on E , and suppose $|f_n(x)| \leq M_n$ ($x \in E, n = 1, 2, 3, \dots$) then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

Q8)

- a) If K is a compact metric space, if $f_n \in C(k)$ for $n = 1, 2, 3, \dots$ and if $\{f_n\}$ is pointwist bounded and equicontinuous on K , then prove that (1) $\{f_n\}$ is uniformly bounded on K (2) $\{f_n\}$ is contains a uniformly convergent subsequence.
- b) If K is a compact metric space, if $f_n \in C(k)$ for $n = 1, 2, 3, \dots$ and if $\{f_n\}$ converges uniformly on K , then prove that $\{f_n\}$ is equicontinuous on K .

Q9)

- a) If f is measurable, then prove that $|f|$ is measurable.
- b) Let f and g are measurable real-valued functions defined on X , let F be real and continuous on \mathbb{R}^2 , and put $h(x) = F(f(x), g(x))$, ($x \in X$) then prove that h is measurable.

Q10)

- a) If $f \in \mathcal{L}(\mu)$ on E , then prove that $|f| \in \mathcal{L}(\mu)$ on E

$$\text{and } \left| \int_E f d\mu \right| \leq \int_E |f| d\mu.$$

- b) Suppose that $f = f_1 + f_2$, where $f_i \in \mathcal{L}(\mu)$ on E ($i = 1, 2, 3, \dots$), then prove that $f \in \mathcal{L}(\mu)$ and

$$\int_E f d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu.$$



M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2019**(First Year)****MATHEMATICS****Complex Analysis and Special Functions and Partial Differential Equations****Time : 3 Hours****Maximum Marks : 70**

Answer any Five of the following questions,
selecting at least two questions from each section.
All questions carry equal marks.

SECTION - A**Q1)**

a) Prove that

$$(1 - 2xz + z^2)^{1/2} = \sum_{n=0}^{\infty} z^n p_n(x), \quad |x| \leq 1, |z| < 1.$$

b) If $u_n = \int_{-1}^1 x^{-1} p_n(x) p_{n-1}(x) dx$, then show that
 $nu_n + (n-1)u_{n-1} = 2$, hence evaluate u_n .

Q2)

a) Show that

$$i) \int_0^x x^{-n} J_{n+1}(x) dx = \frac{1}{2^n \Gamma(n+1)}, \quad n > 1.$$

$$ii) \int_0^{\infty} x^{-n} J_{n+1}(x) dx = \frac{1}{2^n \Gamma(n+1)}, \quad n > -\frac{1}{2}.$$

b) Show that

$$i) \int_0^x x^3 J_0(x) dx = x^3 J_1(x) - 2x^2 J_2(x)$$

$$ii) \int_0^1 x^3 J_0(x) dx = 2J_0(1) - 3J_1(1)$$

- Q3)** a) Solve $(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$.
 b) Solve $z(y + z)dx + z(t - x)dy + y(x - t)dz + y(y + z)dt = 0$.

- Q4)** a) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$.
 b) $(D^2 + 2DD' + D'^2)z = 2 \cos y - x \sin y$.

- Q5)** a) Solve $(D + D' - 1)(D + 2D' - 3)z = 4 + 3x + 6y$.
 b) Solve $(r - t)xy - s(x^2 - y^2) = qx - py$ by Monge's method.

SECTION - B

Q6)

- a) Prove that $\|z\| - \|w\| \leq \|z - w\|$ and give necessary and sufficient conditions for equality.
 b) Let $Z = \text{cis } \frac{2\pi}{n}$ for an integer $n \geq 2$. Show that $1 + z + \dots + z^{n-1} = 0$.

Q7)

- a) If $\sum_{n=0}^{\infty} a_n (z - a)^n$ is a given power series with radius of convergence R , then prove that $R = \lim |a_n / a_{n+1}|$ if this exist.
 b) Let G be either the whole plane C or some open disk. If $u : G \rightarrow R$ is a harmonic function, then show that u harmonic conjugate.

- Q8)** a) State and prove Cauchy's integral formula in second version.

- b) Evaluate $\int_{\gamma} \frac{dz}{z^2 + 1}$ where $\gamma(\theta) = 2|\cos 2\theta|e^{i\theta}$ for $0 \leq \theta \leq 2\pi$.

Q9) a) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$.

b) State and prove general version of Rouché's theorem for curves other than circle in G.

Q10) a) State and prove Maximum Modulus theorem.

b) Evaluate $\int_0^{\infty} \frac{x^2 dx}{x^4 + x^2 + 1}$.



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DM04

M.Sc. DEGREE EXAMINATION, JUNE/JULY - 2019

(First Year)

MATHEMATICS

Theory of Ordinary Differential Equations

Time : 3 Hours

Maximum Marks : 70

Answer any Five of the following questions.

All questions carry equal marks.

Q1)

- a) State and prove existence theorem.
- b) Let ϕ_1, \dots, ϕ_n be n solutions of $L(y) = 0$ on an interval I , and let x_0 be any point in I . Then prove that
- $$W(\phi_1, \dots, \phi_n)(x) = \left| \exp \left[-\int_{x_0}^x a_1(t) dt \right] W(\phi_1, \dots, \phi_n)(x_0) \right|$$

Q2)

- a) Verify the function ϕ_1 satisfies the equation $x^2 y'' - 7xy' + 15xy = 0$, $\phi_1(x) = x^2$, ($x > 0$) and find a second independent solution.
- b) State and prove existence theorem for Analytic coefficients.

Q3)

- a) Find all real - valued solutions of $y' = \frac{e^{x-y}}{1+e^x}$.
- b) Let M, N be two real valued functions which has continuous first partial derivatives on some rectangle $R : |x - x_0| \leq a, |y - y_0| \leq b$. Then show that $M(x,y) + N(x,y) y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Q4)

- a) Show that a function ϕ is a solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on I if and only if it is a solution of the integral equation

$$y = y_0 + \int_{x_0}^x f(t,y) dt.$$

- b) Find the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for $y' = x^2 + y^2$, $y(0) = 0$.

Q5)

- a) Find the solution $\varphi(x)$ of $y'' = 1 + (y')^2$ which satisfies $\varphi(0) = 0$, $\varphi'(0) = 0$.

- b) Suppose that f is a continuous function on an interval $|x - x_0| \leq a$. Show that the solution φ of the initial value problem $y'' = f(x)$, $y(x_0) = \alpha$, $y'(x_0) = \beta$ can be written as

$$\varphi(x) = \alpha + \beta(x - x_0) + \int_{x_0}^x |x - t| f(t) dt.$$

Q6)

- a) State and prove local existence theorem.
b) Consider system where $y_1' = ay_1 + by_2$, $y_2' = -by_1 + ay_2$, a, b are real constants,

(1) If $\phi = (\phi_1, \phi_2)$ is any solution with values in \mathbb{R}_2 , show that $\|\phi(x)\| = \|\phi(0)\| e^{\alpha x}$, where $\|\phi(x)\| = [\phi_1^2(x) + \phi_2^2(x)]^{1/2}$. (2) Verify that the solution satisfies $\phi(0) = (1, 0)$ is given by

$$\phi(x) = e^{\alpha x} (\cos bx, -\sin bx).$$

Q7)

- a) By reduction to linear equation solve the Riccati's equation $y' = -2y^2 - 5y - 2$.
- b) Find the greens function of the boundary value problem $y'' = -f(x), y(0) = 0, y(1) = 0$.

Q8)

- a) Show that if z_1, z_2, z_3 are any four different solutions of the Riccati equation $y' + a(x)y + b(x)y^2 + c(x) = 0$, then show that $\frac{y - y_2}{y - y_1} = \frac{y_3 - y_1}{y_3 - y_2}$.
- b) Find the functions $z(x), k(x), m(x)$ such that $z(x)[x^2 y'' - 2xy' + 2y] = \frac{d}{dx} (k(x) y' + m(x)y)$ and hence solve $x^2 y'' - 2xy' + 2y = 0, x > 0$.

Q9)

- a) State and prove Sturm separation theorem.
- b) Solve $x^2 y'' - 2xy' + (2+x^2)y = 0, x > 0$.

Q10)

- a) State and prove Gronwall's inequality.
- b) Discuss the oscilation of Bessel equation $x^2 y'' - xy' + (x^2 - n^2)y = 0$.

